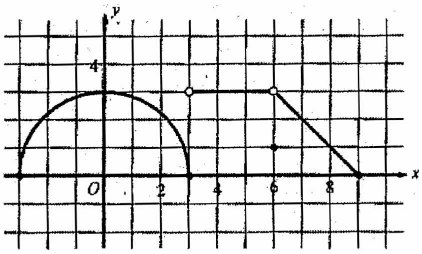
CH2 TEST

Chapter 2 Additional Practice Questions.

Example 2 □ The graph of the function f is shown in the figure below. Find the limit or value of the function at a given point.

(a) $\lim_{x \to 3^-} f(x)$ (b) $\lim_{x \to 3^+} f(x)$ (c) $\lim_{x \to 3} f(x)$

(d) $\lim_{x \to 6} f(x)$ (e) $f(3)$ (f) $f(6)$



Graph of $y = f(x)$

Solution □ (a) $\lim_{x \to 3^-} f(x) = 0$

(b) $\lim_{x \to 3^+} f(x) = 3$

(c) $\lim_{x \to 3} f(x)$ does not exists since $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$.

(d) $\lim_{x \to 6} f(x) = 3$, because $\lim_{x \to 6^-} f(x) = 3 = \lim_{x \to 6^+} f(x)$.

(e) $f(3) = 0$

(f) $f(6) = 1$

4. Let f be a function given by $f(x) = \begin{cases} 3 - x^2, & \text{if } x < 0 \\ 2 - x, & \text{if } 0 \le x < 2 \\ \sqrt{x - 2}, & \text{if } x > 2 \end{cases}$.

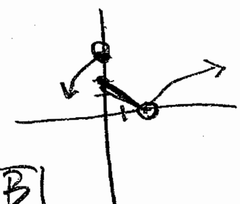
Which of the following statements are true about f?

I. $\lim_{x \to 0} f(x) = 2$

II. $\lim_{x \to 2} f(x) = 0$

III. $\lim_{x \to 1} f(x) = \lim_{x \to 6} f(x)$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III



$\boxed{B}$

2. $\lim_{x \to 0} \frac{\sin 3x}{\sin 2x} =$

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{3}{2}$ (D) 6 (E) nonexistent

$\frac{3x}{3x} \cdot \frac{2x}{2x} \cdot \frac{\sin 3x}{\sin 2x}$

$\frac{3x}{2x} = \boxed{\frac{3}{2}}$

10. Find $\lim_{x \to 0} \frac{f(x) - g(x)}{\sqrt{g(x) + 7}}$, if $\lim_{x \to 0} f(x) = 2$ and $\lim_{x \to 0} g(x) = -3$.

$= \boxed{\frac{5}{2}}$

lim for all
x→

so $\frac{2=3}{\sqrt{4}} = \boxed{\frac{5}{2}}$

1. Let f be a function defined by $f(x) = \begin{cases} \frac{x^2 - a^2}{x - a}, & \text{if } x \neq a \\ 4, & \text{if } x = a \end{cases}$. If f is continuous for all real numbers x, what is the value of a?

(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) 2 (E) 4

$\frac{x^2 - a^2}{x - a} = 4$

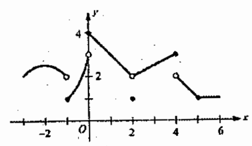
$\frac{\cancel{(x-a)}(x+a)}{\cancel{x-a}} = 4$

$x + a = 4$

but $x = a$

so $a = 2$

$\boxed{D}$



2. The graph of a function f is shown above. If $\lim_{x \to a} f(x)$ exists and f is not continuous at $x = a$, then $a =$

(A) -1 (B) 0 (C) 2 (D) 4 (E) 5

$\boxed{C}$

1) Find the equation of the tangent and normal to $y = 2x^2 - 1$ at $x = 1$.

2) Suppose $4x^2 - 1 \leq f(x) \leq x - 1$ for $-1 < x < 1$. Find the $\lim_{x \to 0} f(x)$.

3) Find the average rate of change of $y = 1 + \cos x$ on $[0, \pi]$

4) Use IVT to show that $2 = x^3 - 4x + 1$ has at least 1 solution. Find the solution(s).

Ans

① P Q $(1, 1)$ $(x, 2x^2 - 1)$

$\lim_{x \to 1} \frac{2x^2 - 1 - 1}{x - 1} = \lim_{x \to 1} \frac{2x^2 - 2}{x - 1}$

$= \lim_{x \to 1} \frac{2(x^2 - 1)}{x - 1} = \frac{2\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$\lim_{x \to 1} 2(x+1) = \boxed{4}$

Thus $m = 4$

$y = 4x + b$

$1 = 4(1) + b$

$-3 = b$

$\boxed{y = 4x - 3}$

② @ $x = 0$

$4(0)^2 - 1 = -1$ $= 0 - 1 = -1$

Thus $\boxed{\lim_{x \to 0} f(x) = -1}$ by squeez thm

③ $f(0) = 1 + \cos 0 = 1 + 1 = 2$

$f(\pi) = 1 + \cos \pi = 1 + (-1) = 0$

ave $= \frac{f(\pi) - f(0)}{\pi - 0} = \frac{0 - 2}{\pi} = \boxed{\frac{-2}{\pi}}$

④ $0 = x^3 - 4x + 1 - 2$

$0 = x^3 - 4x - 1$

If you choose $x = 1$ then $1^3 - 4(1) - 1 = (-)$

If you choose $x = 5$ then $5^3 - 4(5) - 1 = (+)$

* Since there is a sign change & it's continuous, then IVT guarantees a root.

use calc $x = -1.86$, $-.2541$, 2.115