AP Calculus Chapter 3 Test - Practice Problems

1. Water is flowing out of a cone at a rate of 50 cm^3/sec. The height of the cone is 20 cm and the diameter is 6 cm.

a) Find the rate of change in the height of the cone when the radius is 1 cm.

b) Find the rate of change in the radius of the cone when the radius is 1 cm.

2. Implicitly differentiate: $2x^3 + y^3 = 4x^2y$

b) Find the slope of the tangent line(s) of the curve at x = 1.

3. Suppose the motion of a particle is given by $s(t) = t^3 + 5t^2 - 7t + 2$

a) find the velocity function

b) find the acceleration function

c) When is the particle moving to the left? Right?

d) When does the particle change direction?

e) When is the particle slowing down? Speeding up?

f) Sketch the motion of the particle.

4. Suppose $f(x) = [2g(x)]^4$, find f'(x) in terms of g'(x)

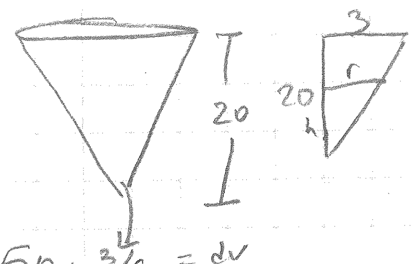
5. Suppose $f(x) = \begin{cases} 3ax^2 + bx & \text{when } x > 1 \\ ax^3 + 2bx + 3 & \text{when } x \leq 1 \end{cases}$ and f(x) is continuous and differentiable.

Find a and b.

6. The radius of a sphere is measured with an error of .2%. Estimate the percent error in the volume calculation.

7. Two cars leave the same point at the same time. One heads south at 40 km/h and the other east at 50 km/h. After 3 hours, how fast is the distance between the cars increasing?

① $r=3$



$\frac{r}{h} = \frac{3}{20}$

$-50\ cm^3/s = \frac{dV}{dt}$

ⓐ $V = \frac{1}{3}\pi r^2 h$, need $\frac{dh}{dt}$ so $r = \frac{3}{20}h$

$V = \frac{1}{3}\pi\left(\frac{3}{20}h\right)^2 \cdot h$

$V = \frac{1}{3}\pi \frac{9}{400}h^3 = \frac{3}{400}\pi h^3$

$\frac{dV}{dt} = \frac{9}{400}\pi h^2 \frac{dh}{dt}$ → when $r=1$, $h = \frac{20}{3}$

$-50 = \frac{9}{400}\pi\left(\frac{20}{3}\right)^2 \frac{dh}{dt}$

$\boxed{-15.92\ cm/s = \frac{-50}{\pi} = \frac{dh}{dt}}$

ⓑ $V = \frac{1}{3}\pi r^2 h$, need $\frac{dr}{dt}$

$V = \frac{1}{3}\pi r^2\left(\frac{20}{3}r\right)$ so $h = \frac{20}{3}r$

$V = \frac{20}{9}\pi r^3$

$\frac{dV}{dt} = \frac{20}{3}\pi r^2 \frac{dr}{dt}$ so when $r=1$

$-50 = \frac{20}{3}\pi(1^2)\frac{dr}{dt}$

$\frac{-15}{2\pi} = \boxed{\frac{dr}{dt} \approx -2.388\ cm/s}$

② $2x^3 + y^3 = 4x^2y$

$6x^2 + 3y^2y' = 4x^2 \cdot y' + y \cdot 8x$

$3y^2y' - 4x^2y' = 8xy - 6x^2$

$y'(3y^2 - 4x^2) = 8xy - 6x^2$

$y' = \boxed{\frac{dy}{dx} = \frac{8xy - 6x^2}{3y^2 - 4x^2}}$

ⓑ @ $x=1$, $2(1)^3 + y^3 = 4(1^2)y$

$2 + y^3 = 4y$

$y^3 - 4y + 2 = 0$

$y = -2.21 \quad y = .539 \quad y = 1.675$

so $\frac{dy}{dx} = m_1 = \frac{8(1)(-2.21) - 6(1^2)}{3(-2.21) - 4(1^2)} =$

$m_2 = \frac{8(1)(.539) - 6(1^2)}{3(.539) - 4(1^2)} =$

$m_3 = \frac{8(1)(1.675) - 6(1^2)}{3(1.675) - 4(1^2)} =$

③ $s(t) = t^3 + 5t^2 - 7t + 2$

ⓐ $v(t) = 3t^2 - 10t - 7$ ⓑ $a(t) = 6t - 10$

ⓒ left when $v<0$ $\quad 0 = 3t^2 - 10t - 7$

right when $v>0$ $\quad$ USE QUAD

+++ | ---- | ++++

$-.5941 \quad 3.927$

left from $[0, 3.927)$

right $x > 3.927$

ⓓ Changes direction when v changes sign thus @ $x = 3.927$

ⓔ speeding up $v(t)$ $a(t)$ same sign

slowing down $v(t)$ & $a(t)$ opposite signs

$a(t)$ --- | +++ $\frac{5}{3}$

$6t - 10 = 0$

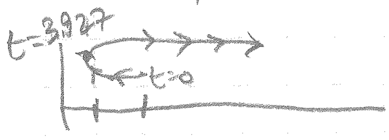
$t = \frac{10}{6} = \frac{5}{3} \approx 1.667$

$[0, 1.667] \rightarrow$ both $(-)$ speeding up

$x > 3.927$ both $(+)$ " "

$[1.667, 3.927]$ opposite, so slowing down

ⓕ



(4) $f(x) = [2g(x)]^4$

$f'(x) = 4[2g(x)]^3 \cdot 2g'(x) = 8[2g(x)]^3 \cdot g'(x)$

(5) @ $x=1 \rightarrow 3a+b = a+2b+3$ $\quad 2a-b=3$

plug 1 into x → Thus → $3a-b=0$ Solve Thus $\boxed{\begin{matrix} a=-3 \\ b=-9 \end{matrix}}$

$f'(x) = \begin{cases} 6ax+b \\ 3ax^2+2b \end{cases}$ @ $x=1$ $\quad 6a+b=3a+2b$

$3a-b=0$

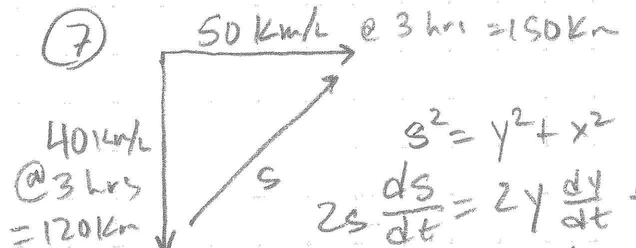
(6) $\frac{dr}{r} = \pm.002$ $\quad V = \frac{4}{3}\pi r^3$ $\quad dV = 4\pi r^2 dr$ → So $\frac{dV}{V} = \frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3} = \frac{3}{r} dr$

$-.002 < \frac{dr}{r} < .002$ $\qquad \frac{1}{3}\frac{dV}{V} = \frac{dr}{r}$

$-.002 < \frac{1}{3}\frac{dV}{V} < .002$

$.006 < \frac{dV}{V} < .006$ → $\boxed{\frac{dV}{V} = .6\%}$

(7)



$s^2 = y^2 + x^2$

$2s\frac{ds}{dt} = 2y\frac{dy}{dt} + 2x\frac{dx}{dt}$ → @ 3 hrs $s^2 = 120^2 + 150^2$

$s = 192.09$

$2(192.09)\frac{ds}{dt} = 2(120)(40) + 2(150)(50)$

$\boxed{\frac{ds}{dt} = 64.03 \text{ km/h}}$