AP Calculus Chapter 3 Review Test 1

1. Find the average rate of change of $f(x) = x^2$ on [-1, 2].

2. Use definition of derivative to find $f'(x)$ for $f(x) = x^2 + 3x + 2$.

3. Consider a particle whose motion is represented by $s(t) = 3t^2 - 2t + 1$, where $t \geq 0$.
    a) Find the equation of the velocity.
    b) What is the acceleration equation? What is a(4)?
    c) Find the position at t = 4.
    d) Find the distance travelled by the particle in the first 2 seconds.

4. Sketch a graph of a function with the following properties:
    $f(0) = 4$, $f'(0) = 0$, $f'(-4) = 1$, $f'(4) = 0$, $f(2) = -1$, $f'(6) = 1$

5. Suppose at x = 2, f(2) = 5, f'(2) = 12, g(2) = -1, g'(2) = 3. Find the derivative of $f(x)\cdot g(x)$

6. Find the tangent line to $f(x) = \dfrac{x}{x^2+1}$ at (1, ½).

7. Where does $f(x) = x^2 + x$ have a horizontal tangent?

8. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for $y = 2x \cos x$

Sample Problems - Review?

① Find ave rate of Change of $f(x)=x^2$ on $[-1,2]$.

Soln $\frac{f(2)-f(-1)}{2--1} = \frac{4-1}{2+1} = \frac{3}{3} = \boxed{1}$

② Use defn of derivative to find $f'(x)$ for $f(x)=x^2+3x+2$

Soln $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+3(x+h)+2-(x^2+3x+2)}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2+\cancel{3x}+3h+\cancel{2}\cancel{-x^2}\cancel{-3x}\cancel{-2}}{h} = \lim_{h\to 0} \frac{2xh+h^2+3h}{h}$

$= \lim_{h\to 0} 2x+h+3 = \boxed{2x+3}$

③ Consider a particle whose motion is represented by $s(t)=3t^2-2t+1$ where $t\geq 0$.

ⓐ Find the equation of the velocity
ⓑ What is the acceleration equation? What is $a(4)$?
ⓒ Find the position at $t=4$.
ⓓ Find the distance traveled by the particle in the first 2 seconds.

Soln ⓐ $v(t)=6t-2$ ⓑ $a(t)=6$ $a(4)=6$

ⓒ $s(4) = 3(4)^2-2(4)+1 = 3(16)-8+1 = \boxed{41}$

ⓓ velocity is (-) from $t=0$ to $t=\frac{1}{3}$ ⟶ $6t-2=0$ } $6t=2$

$t=\frac{2}{6}=\frac{1}{3}$

Thus changes direction @ $t=\frac{1}{3}$

So distices = dist traveled from 0 to $\frac{1}{3}$ + dis from $\frac{1}{3}$ to 2

Thus $d = |s(\tfrac{1}{3})-s(0)| + s(2)-s(\tfrac{1}{3})$

$= |\tfrac{2}{3}-1| + 9-\tfrac{2}{3} = \boxed{8\tfrac{2}{3}}$

④ Know how to do problems like # 1-11 on p.131-132 & 19-20 on p.120

⑤ Suppose at $x=2$ $f(2)=5$ $f'(2)=12$ $g(2)=-1$ $g'(2)=3$
Find the derivative of $f(x)\cdot g(x)$

Soln
deriv $f(x)\cdot g(x) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$ @ $x=2$

$= 5\cdot(3) + (-1)(12) = 15-12 = \boxed{3}$

(6) Find the tangent line to $f(x) = \frac{x}{x^2+1}$ @ $(1, \frac{1}{2})$

Soln $f'(x) = \frac{(x^2+1)\cdot 1 - (x)(2x)}{(x^2+1)^2} = m$ @ $x=1$ $= \frac{(2)-(2)}{4} = \frac{0}{4} = 0$

Thus $y = mx+b$ $\quad y = 0x+b$ $\quad b = \frac{1}{2}$ so $\boxed{y = \frac{1}{2}}$

$\frac{1}{2} = (0)(1) + b$

(7) Where does $f(x) = x^2+x$ have a horiz tangent?

Soln

$f'(x) = 2x+1$ $\quad 0 = 2x+1$

$\boxed{-\frac{1}{2} = x}$

(8) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $y = 2x\cos x$

Soln $\frac{dy}{dx} = 2x(-\sin x) + \cos x \cdot 2 = -2x\sin x + 2\cos x$

$\frac{d^2y}{dx^2} = -2x\cdot\cos x + (\sin x)(-2) + 2(-\sin x)$

$= -2x\cos x - 2\sin x - 2\sin x = \boxed{-2x\cos x - 4\sin x}$

(9) Be able to determine intervals of continuity & the defn of continuity and when a function is differentiable.

(10) Know how to use product and quotient rule to find derivative of functions.

(11) Know how to use nDeriv on your calc
nDeriv (equation, x, point)

(12) Review when a function is increasing/decreasing

(13) Given a graph, be able to "draw" the derivative
← you have several of these problems!