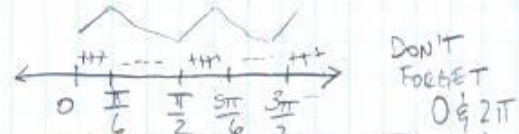


4-1 to 4-4 Review Answers

② a) $f(x) = \sin x + \cos^3 x$ $[0, 2\pi]$
 $f(x) = \cos x + 2(\cos x)(\sin x)$ $[0, 2\pi]$

$0 = \cos x - 2\cos x \sin x$
 $0 = \cos x (1 - 2\sin x)$
 Need $\cos x = 0 \leftarrow @ \frac{\pi}{2} \& \frac{3\pi}{2}$
 Need $1 - 2\sin x = 0 \leftarrow @ \frac{\pi}{6} \& \frac{5\pi}{6}$

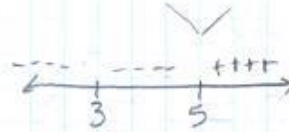
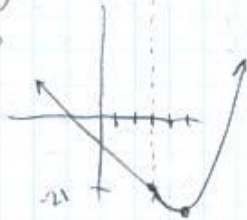
Decreasing $(\frac{\pi}{6}, \frac{\pi}{2}), (\frac{5\pi}{6}, \frac{3\pi}{2})$
 Increasing $[0, \frac{\pi}{6}), (\frac{\pi}{2}, \frac{5\pi}{6}), (\frac{3\pi}{2}, 2\pi]$



Rel MAX @ $\frac{\pi}{6} \& \frac{5\pi}{6} \& 2\pi$
 Rel Min @ $0, \frac{\pi}{2}, \frac{3\pi}{2}$
 ↳ Because slopes change

b) $f(x) = \begin{cases} -x-18 & x \leq 3 \\ x^2-10x & x > 3 \end{cases}$

$f'(x) = -1 \quad x \leq 3$
 $f'(x) = 2x-10 \quad x > 3$ CRIT PTS $3 \& 5$
 $2x-10=0$
 $2x=10$
 $x=5$



Decreasing $(-\infty, 5)$
 Increasing $(5, \infty)$

REL MIN $x=5$ since changes slope

② a) $f(x) = \frac{1}{3}x^3 - x^2 - 3x$
 $f'(x) = x^2 - 2x - 3 = 0$
 $(x-3)(x+1) = 0$

CRIT PTS @ $x=3, -1$
 $(3, -9) \quad (-1, 3)$

Inflection @ $x=1$

$f''(x) = 2x-2$
 $f''(3) = 6-2 > 0 \rightarrow$ Rel Min
 $f''(-1) = -2 < 0 \rightarrow$ Rel Max
 $f'(x) = 0 = 2x-2$
 $2x = 2 \quad x=1$

Thus Rel Min @ $(3, -9)$ Rel Max $(-1, 3)$



Increasing $x < -1 \& x > 3$
 decreasing $-1 < x < 3$



Concave up $x > 1$
 Concave down $x < 1$

$$f(x) = x(x-1)^2$$

$$= x(x^2 - 2x + 1)$$

$$f(x) = x^3 - 2x^2 + x$$

$$f'(x) = 3x^2 - 4x + 1 = 0$$

$$(3x-1)(x-1) = 0$$

$$x = 1 \frac{1}{3}, 1$$



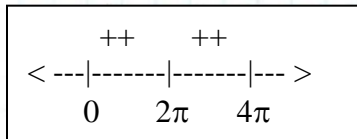
CRIT PTS $x = 1 \frac{1}{3}$ Local Max @ $\frac{1}{3}$
 Min @ 1
 Increasing $(-\infty, \frac{1}{3})$ & $(1, \infty)$ Concave up $(\frac{2}{3}, \infty)$
 Decreasing $(\frac{1}{3}, 1)$ Concave Down $(-\infty, \frac{2}{3})$
 $f''(x) = 6x - 4 = 0$
 $x = \frac{4}{6}$ Possible Infr @ $\frac{2}{3}$
 $f''(1) > 0 \curvearrowright$ MIN $f'(\frac{1}{3}) < 0 \curvearrowright$ MAX

$$f(x) = x - \sin x \quad [0, 4\pi]$$

$$f'(x) = 1 - \cos x = 0$$

$$\cos x = 1 \quad [0, 4\pi]$$

CRIT PTS @ $0, 2\pi, 4\pi$



All (+) on Interval

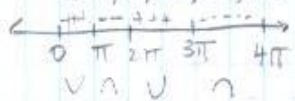
CRIT PTS $x = 0, 2\pi, 4\pi$
 * No Rel extrema
 Inflection Pts @ $x = \pi, 2\pi, 3\pi$

Increasing $(0, 4\pi)$
 Decreasing: none

Concavity
 UP $(0, \pi), (2\pi, 3\pi)$
 Down $(\pi, 2\pi)$ & $(3\pi, 4\pi)$

$$f''(x) = \sin x = 0 \quad [0, 4\pi]$$

@ $0, \pi, 2\pi, 3\pi, 4\pi$ ← Possible inflection



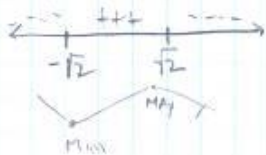
cannot determine 0 & 4pi since endpoints of interval

$$f(x) = \frac{x}{x^2+2}$$

$$f(x) = \frac{(x^2+2)(1) - x(2x)}{(x^2+2)^2} = \frac{x^2+2-2x^2}{(x^2+2)^2} = \frac{-x^2+2}{(x^2+2)^2}$$

$$f'(x) = 0 \quad @ \quad -x^2+2 = 0 \quad x = \pm\sqrt{2}$$

* None at denominator since $x^2 \geq 0$



CRIT PTS @ $x = \pm\sqrt{2}$
 Rel MAX $\sqrt{2}$ Increasing $(\sqrt{2}, \sqrt{2})$
 Rel Min $-\sqrt{2}$ Decreasing $(-\infty, -\sqrt{2})$
 Inflection $\pm\sqrt{6}, 0$
 Concave UP $(\sqrt{6}, 2)$ & $(\sqrt{6}, \infty)$
 Concave Down $(-\infty, -\sqrt{6})$ & $(0, \sqrt{6})$

$$f''(x) = \frac{2x(x^2-6)}{(x^2+2)^3} = 0$$

$$x = 0 \text{ & } \pm\sqrt{6}$$

4. $f(b) - f(a) = f'(c)(b - a)$

Solve for: $f(b) = 0, f(a) = 0, f'(c) = 2c - 1$ So $0 = (2c - 1)(7)$ Thus $c = \frac{1}{2}$

5. In class.