AP Calculus Semester 1 Final REVIEW

NO CALCULATOR MAY BE USED IN THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems. After examining the form of the choices, decide which is the best of the choices given.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^3 (3x^2 - 4x)\,dx =$

   (A) 8 (B) 9 (C) 10
   (D) 12 (E) 62

2. If $f(x) = x\sqrt{4x-1}$, then $f'(x)$ is

   (A) $\frac{6x-1}{\sqrt{4x-1}}$. (B) $\frac{2x}{\sqrt{4x-1}}$. (C) $\frac{1}{\sqrt{4x-1}}$.
   (D) $\frac{-6x+2}{\sqrt{4x-1}}$. (E) $\frac{9x-2}{2\sqrt{4x-1}}$.

3. If $\int_a^b g(x)\,dx = 4a + b$, then $\int_a^b (g(x) + 7)\,dx =$

   (A) $8b - 11a$. (B) $8b + 11a$. (C) $8b - 3a$.
   (D) $7b - 7a$. (E) $4a + b + 7$.

4. If $f(x) = -x^5 + x + \frac{1}{x^2}$, then $f'(-1) =$

   (A) 8. (B) 2. (C) −2.
   (D) −3. (E) −8.

5. $y = 5x^4 - 24x^3 + 24x^2 + 17$ is concave down for

   (A) $x < 0$. (B) $x > 0$.
   (C) $x < -2$ or $x > -\frac{2}{5}$. (D) $x < \frac{2}{5}$ or $x > 2$.
   (E) $\frac{2}{5} < x < 2$.

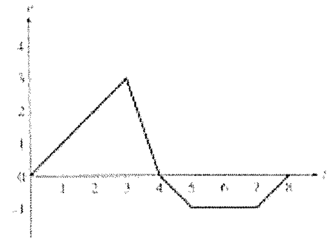
6. $\frac{1}{3}\int e^{t/3}\,dt =$

   (A) $e^t + C$ (B) $3e^{t/3} + C$ (C) $e^{t/3} + C$
   (D) $\frac{1}{3}e^{t/3} + C$ (E) $e^{-2/3t} + C$

7. $\frac{d}{dx}\cos^3 x^2 =$

   (A) $6x\cos^2 x^2$ (B) $\sin^3 x^2$
   (C) $6x\sin x^2 \cos^2 x^2$ (D) $-3\sin x^2 \cos^2 x^2$
   (E) $-6x\sin x^2 \cos^2 x^2$

Questions 8–9 refer to the following situation.



A spider begins to crawl up a vertical blade of grass at time $t = 0$. The velocity v of the spider at time t, $0 \le t \le 8$, is given by the function whose graph is shown.

8. At what value of t does the spider change direction?

(A) 3 (B) 4 (C) 5

(D) 7 (E) 8

9. What is the total distance the spider traveled from $t = 0$ to $t = 8$?

(A) 3 (B) 8 (C) 9

(D) 10 (E) 15

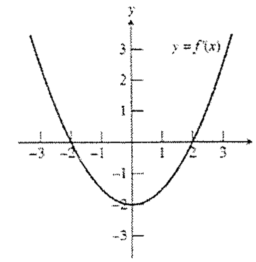
10. An equation of the line tangent to the graph of $y = \cos 3x$ at $x = \pi/6$ is

(A) $y = 3\left(x - \frac{\pi}{6}\right)$ (B) $y = -\left(x - \frac{\pi}{6}\right)$

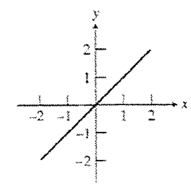
(C) $y = -3\left(x - \frac{\pi}{6}\right)$ (D) $y - 1 = -\left(x - \frac{\pi}{6}\right)$

(E) $y - 1 = -2\left(x - \frac{\pi}{6}\right)$

11. The graph of the derivative of f is shown in the figure below. Which of the following could be the graph of f?



(A)



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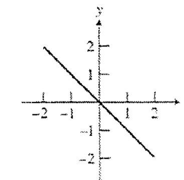
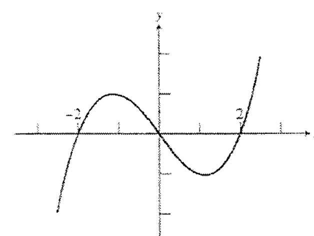
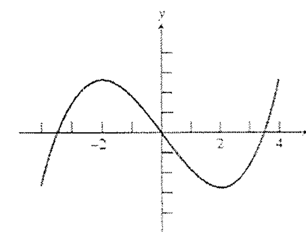


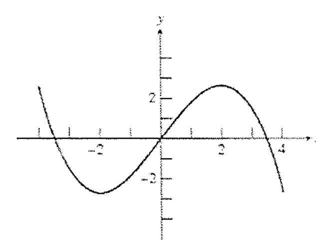
(C)



(D)



(E)



12. At what point on the graph of $y = \frac{1}{2}x^2 - \frac{3}{2}$ is the tangent line parallel to the line $4x - 8y = 5$?

(A) $\left(\frac{1}{2}, -\frac{3}{8}\right)$ (B) $\left(\frac{1}{2}, -\frac{11}{8}\right)$ (C) $\left(2, \frac{3}{8}\right)$

(D) $\left(2, \frac{1}{2}\right)$ (E) $\left(-\frac{1}{2}, -\frac{11}{8}\right)$

13. Let f be a function defined for all real numbers x. If $f'(x) = \frac{|9 - x^2|}{x - 3}$, then f is decreasing on the interval

(A) $(-\infty, 3)$. (B) $(-\infty, \infty)$. (C) $(-3, 6)$.

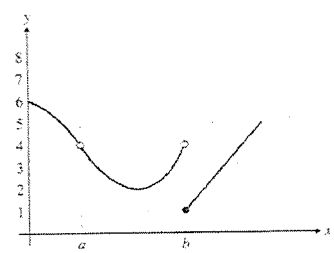
(D) $(-3, \infty)$. (E) $(3, \infty)$.

14. Let f be a differentiable function such that $f(5) = 3$ and $f'(5) = 2$. If the tangent line to the graph of f at $x = 5$ is used to find an approximation to a zero of f, that approximation is

(A) 6.5. (B) 4.3. (C) 3.5.

(D) 0.5. (E) 0.3.

15. The graph of the function f is shown. Which of the following statements about f is true?



(A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$ (B) $\lim_{x\to a} f(x) = 4$

(C) $\lim_{x\to b} f(x) = 4$ (D) $\lim_{x\to b} f(x) = 1$

17. If $x^2 = 25 - y^2$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 4)$?

(A) $\frac{-25}{64}$ (B) $\frac{-7}{64}$ (C) $\frac{7}{64}$

(D) $\frac{25}{64}$ (E) $\frac{4}{3}$

24. The expression $\frac{1}{30}\left(\sin\frac{1}{30} + \sin\frac{2}{30} + \sin\frac{3}{30} + \cdots + \sin\frac{30}{30}\right)$ is a Riemann sum approximation for

(A) $\int_0^1 \sin\frac{x}{30}\,dx$. (B) $\int_0^1 \sin x\,dx$.

(C) $\frac{1}{30}\int_0^1 \sin\frac{x}{30}\,dx$. (D) $\frac{1}{30}\int_0^1 \sin x\,dx$.

(E) $\frac{1}{30}\int_0^{30} \sin x\,dx$.

26. Let $f(x) = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$. For what value of x does $f(x) = 4$?

(A) -2 (B) -1 (C) 1

(D) 2 (E) 4

27. Let $f(x) = \begin{cases} 3x^2 - 5, & x \le 1 \\ 6x + 2, & x > 1 \end{cases}$. Which of the following are true statements about this function?

I. $\lim_{x\to 1} f(x)$ exists.

II. $\lim_{x\to 1} f'(x)$ exists.

III. $f'(1)$ exists.

(A) None (B) II only (C) III only

(D) II and III (E) I, II, and III

28. Let $g(x) = \frac{d}{dx}\int_0^x \sqrt{t^2 + 9}\,dt$. What is $g(-4)$?

(A) -5 (B) -3 (C) 3

(D) 4 (E) 5

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems. After examining the form of the choices, decide which is the best of the choices given.

In this test:

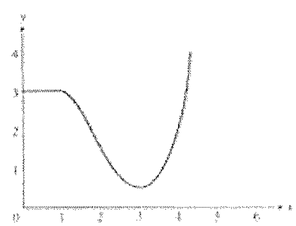
1. The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

30. The graph of the function $y = \frac{1}{3}x^3 - x^2 - 5x + 3\sin x$ changes concavity at $x =$

(A) 3.29. (B) 2.21. (C) 1.34.

(D) 0.41. (E) −0.39.

31. The graph of f is shown. If $\int_1^4 f(x)\,dx = 3.8$ and $F'(x) = f(x)$, then $F(4) - F(0) =$



(A) 0.8. (B) 2.8. (C) 4.8.

(D) 6.8. (E) 8.4.

32. Let f be a function such that $\lim_{h \to 0} \frac{f(7+h) - f(7)}{h} = 4$. Which of the following must be true?

I. f is continuous at $x = 7$.

II. f is differentiable at $x = 7$.

III. The derivative of f is continuous at $x = 7$.

(A) I only (B) II only (C) I and II only

(D) I and III only (E) II and III only

34. Two roads cross at right angles, one running north/south and the other east/west. Eighty feet south of the intersection is an old radio tower. A car traveling at 50 feet per second passes through the intersection heading east. At how many feet per second is the car moving away from the radio tower 3 seconds after it passes through the intersection?

(A) 43.65 (B) 44.12 (C) 44.59

(D) 36.67 (E) 81.76

35. If $y = 3x + 6$, what is the minimum value of x^2y?

(A) −10.125 (B) −5.0625 (C) −1.5

(D) 0 (E) 1.5

38. If the derivative of f is given by $f'(x) = 2e^x - 5x^2$, at which of the following values of x does f have a relative maximum value?

(A) −0.494 (B) 0.259 (C) 1.092

(D) 2.543 (E) 3.310

39. Let $f(x) = \sqrt{2x}$. If the rate of change of f at $x = c$ is four times its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{16}$. (B) $\frac{1}{2\sqrt{2}}$. (C) $\frac{1}{\sqrt{2}}$.

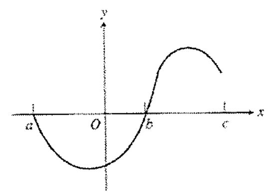
(D) 1. (E) 32.

40. At time $t \geq 0$, the acceleration of a particle that is moving along the x-axis is $a(t) = t + 2\sin t$. At $t = 0$, the velocity of the particle is −4. For what value of t will the velocity of the particle be zero?

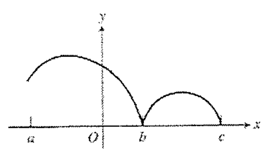
(A) 0 (B) 1.20 (C) 1.78

(D) 2.31 (E) 3.87

41. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown below. Which of the following could be the graph of f?



(A)



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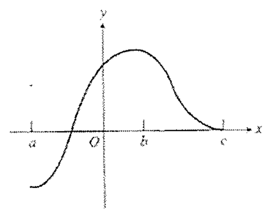

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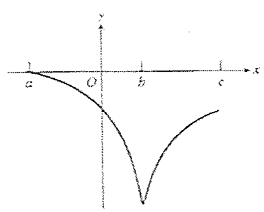
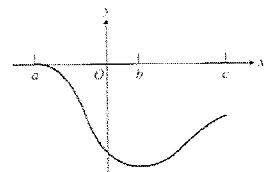


(D)



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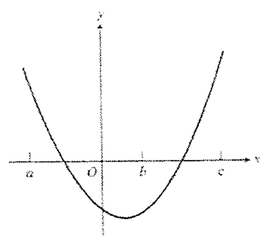


42. A continuous function $f(x)$ has the values shown in the table. What is the value of a trapezoidal approximation of $\int_0^3 f(x)\,dx$ using six equal subintervals?

x	0	0.5	1.0	1.5	2.0	2.5	3.0
$f(x)$	8	5	4	3	3	5	8

(A) 9 (B) 14 (C) 18

(D) 28 (E) 56

43. Which of the following are antiderivatives of $f(x) = 4\sin x\cos x$?

I. $F(x) = -\cos 2x$
II. $F(x) = 2\sin^2 x$
III. $F(x) = -2\cos^2 x$

(A) I only (B) II only (C) III only

(D) I and II (E) I, II, and III

44. Let f be a function such that $f''(x) < 0$ for all x in the closed interval $[3, 4]$, with selected values shown in the table. Which of the following must be true about $f'(3.3)$?

x	3.2	3.3	3.4	3.5
$f(x)$	2.48	2.68	2.86	3.03

(A) $f'(3.3) < 0$ (B) $0 < f'(3.3) < 1.6$

(C) $1.6 < f'(3.3) < 1.8$ (D) $1.8 < f'(3.3) < 2.0$

(E) $f'(3.3) > 2.0$

45. If the function f is defined by $f(x) = \int_0^x -\sin t^2\,dt$ on the closed interval $-1 \le x \le 3$, then f has a local maximum at $x =$

(A) -1.084. (B) 0. (C) 1.772.

(D) 2.171. (E) 2.507.

Calculus AB—Exam 2
Section II, Part A

Time: 45 minutes
Number of problems: 3

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS IN THIS PART OF THE EXAMINATION.

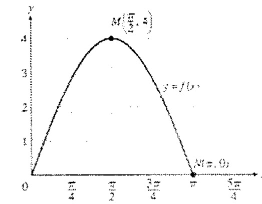
1. A particle moves along the x-axis in such a way that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 4t - 4$. The particle's position $x(t)$ has a value of 1 when $t = 1$.

   (A) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

   (B) For what value(s) of t, $0 \leq t \leq 4$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0, 4]$?

   (C) Find the total distance traveled by the particle from time $t = 0$ until time $t = 4$.

2. Let f be the function given by $f(x) = 4 \sin x$. As shown, the graph of f passes through the point $M(\pi/2, 4)$ and crosses the x-axis at point $N(\pi, 0)$.



   (A) Write an equation for the line passing through points M and N.

   (B) Write an equation for the line tangent to the graph of f at point N. Show the analysis that leads to your equation.

   (C) Find the x-coordinate of the point on the graph of f, between points M and N, at which the line tangent to the graph of f is parallel to line MN.

3. A crate of supplies is dropped from an airplane with a remote-controlled parachute. Let $v(t)$ be the velocity (in meters per second) of the crate at time t seconds, $t \geq 0$. After the parachute opens, the velocity of the crate satisfies the differential equation $dv/dt = -3v - 15$, with initial condition $v(0) = -15$.

   (A) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

   SKIP

   (B) Find the terminal velocity of the crate to the nearest meter per second [terminal velocity is defined as $\lim_{t \to \infty} v(t)$].

   (C) It is safe for the package to land once the crate is dropping no more than 5.3 meters per second. At what time t does the crate reach this speed?

Calculus AB—Exam 2
Section II, Part B

Time: 45 minutes
Number of problems: 3

NO CALCULATOR MAY BE USED IN THIS PART OF THE EXAMINATION.

4. Let f be the function given by $f(x) = 3\sqrt{x-2}$.

   (A) On the axes provided, sketch the graph of f and shade the region R enclosed by the graph of f, the x-axis, and the vertical line $x = 8$.

   (B) Find the area of the region R described in part (A).

   (C) Rather than using the line $x = 8$ as in part (A), consider the line $x = k$, where k can be any number greater than 2. Let $A(k)$ be the area of the region enclosed by the graphs of f, the x-axis, and the vertical line $x = k$. Write an integral expression for $A(k)$.

   (D) Let $A(k)$ be as described in part (C). Find the rate of change of A with respect to k when $k = 8$.

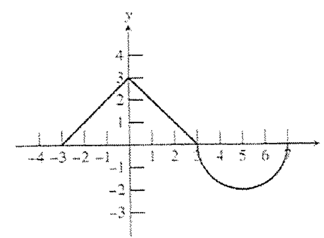
5. Let f be the function given by $f(x) = x^3 - 3x^2 + k$, where k is an arbitrary constant.

   (A) Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values for f in terms of k. Show the analysis that leads to your conclusion.

   (B) For what values of the constant k does f have three distinct real zeros?

   (C) Find the value of k such that the average value of f over the closed interval $[-2, 1]$ is 2.

6. The graph of a function f consists of a semicircle and two line segments, as shown. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.



   (A) Find $g(5)$.

   (B) Find all values of x on the open interval $(-3, 7)$ at which g has relative maximum. Justify your answer.

   (C) Write an equation for the line tangent to the graph of g at $x = 5$.

   (D) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-3, 7)$. Justify your answer.

Calculus AB—Exam 2

Section I

Part A—No Calculator

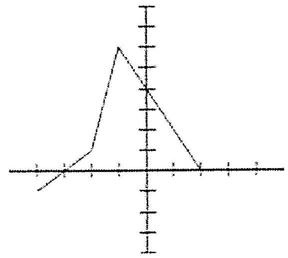
Problem	Answer
1.	(C)
2.	(A)
3.	(C)
4.	(C)
5.	(E)
6.	(C)
7.	(E)
8.	(B)
9.	(C)
10.	(C)
11.	(D)
12.	(B)
13.	(A)
14.	(C)
15.	(B)
16.	(C)
17.	(A)
18.	(B)
19.	(B)
20.	(E)
21.	(A)
22.	(D)
23.	(E)
24.	(B)
25.	(A)
26.	(D)
27.	(B)
28.	(E)

Part B—Calculator Allowed

Problem	Answer
29.	(E)
30.	(B)
31.	(D)
32.	(C)
33.	(B)
34.	(B)
35.	(B)
36.	(B)
37.	(B)
38.	(C)
39.	(A)
40.	(C)
41.	(D)
42.	(B)
43.	(E)
44.	(D)
45.	(E)

2. For $-4 \le t \le 2$ the graph of a function f is shown below. Let $g(x) = \int_0^{\frac{1}{2}x} f(t)\,dt$.

The graph of $f(t)$



a. What is the domain of $g(x)$?

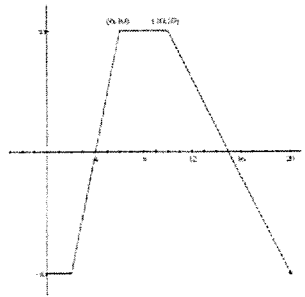
b. Compute, or state that it does not exist, $g(-2), g'(-2), g''(-2)$

c. Find all values of x where $g(x)$ has a relative minimum. Justify your answer.

d. Find all values of x in the open interval $(-8, 4)$ at which the graph of g has a point of inflection.

4. A car is traveling on a straight road with velocity 50 ft/sec at time $t = 0$. For $0 \le t \le 20$, the car's acceleration $a(t)$, in $ft/\sec^2$, is the piecewise linear function defined by the graph below.

$a(t)$



a. Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

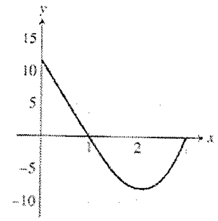
b. Is the speed of the car increasing or decreasing at time $t = 2$? Give a reason for your answer.

c. At what time in the interval $0 < t \le 20$ is the velocity of the car 50 ft/sec?

d. On the time interval $0 \le t \le 20$, what is the car's absolute maximum velocity, in ft/sec? At what time does it occur? Why?

3. Let $F(x) = \int_0^x f(t)\,dt$, where $f(t)$ is the continuous function whose graph is shown.

(A) Where does F achieve its maximum value? Explain.



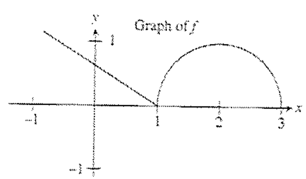
$y = f(x)$

(B) Where does F achieve its minimum value? Explain.

(C) Sketch a possible graph for F on the interval $[0, 3]$.

1. Evaluate $\frac{d}{dx}\int_x^5 2^{t^2}\,dt$.

2. Let f be the continuous function defined on $[-1, 3]$ whose graph, shown below, is comprised of a line segment and semicircle.



If $g(x) = \int_1^x f(1)\,dt$, use the graph of f on the previous page to

(A) Find $g(-1)$, $g(1)$, and $g(3)$

(B) Find intervals on $[-1, 3]$ where g is decreasing.

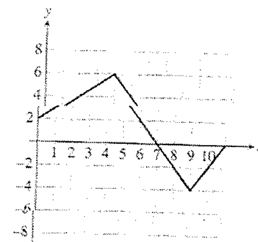
(C) Find intervals on $[-1, 3]$ where the graph of g would be concave downward.

3. Which of the following is the greatest value of x on the interval $[0, 3]$ for which $\int_0^x (t^2 - 5t)\,dt \geq \int_2^{3x} t\,dt$?

(A) 0.56 (B) 0.92 (C) 1.36 (D) 1.57 (E) 1.78

4. Let f be the differentiable function whose graph is shown. The position at time t (in seconds) of a particle moving along a coordinate axis is $s = \int_0^t f(x)\,dx$ meters.

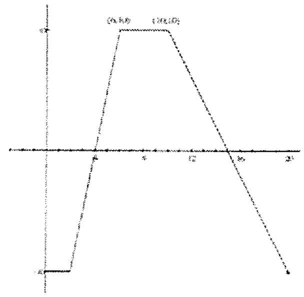
(A) What is the particle's position at time $t = 2$?



(B) At what time during the first 11 seconds does the particle's position have its largest value? Justify your answer.

4. A car is traveling on a straight road with velocity 50 ft/sec at time $t = 0$. For $0 \le t \le 20$, the car's acceleration $a(t)$, in $ft/\sec^2$, is the piecewise linear function defined by the graph below.

$a(t)$



a. Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

b. Is the speed of the car increasing or decreasing at time $t = 2$? Give a reason for your answer.

c. At what time in the interval $0 < t \le 20$ is the velocity of the car 50 ft/sec?

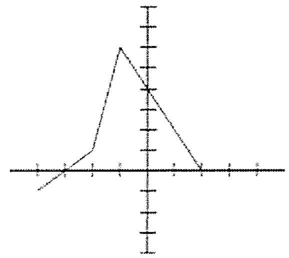
d. On the time interval $0 \le t \le 20$, what is the car's absolute maximum velocity, in ft/sec? At what time does it occur? Why?

---

a. Since $v'(2) = a(2) = -10$, the velocity is decreasing at $t = 2$	$2 \begin{cases} 1: \text{ answer} \\ 1: \text{ reason} \end{cases}$
b. $v(2) = v(0) + \int_0^2 a(t)dt = 50 + -20 = 30$ ft/sec Speed is decreasing since $a(2) < 0$ and $v(2) > 0$	$2 \begin{cases} 1: \text{ decreasing} \\ 1: \text{ reason} \end{cases}$
c. at time $t = 8$ $v(8) = v(0) + \int_0^8 a(t)dt = 50 + 0 = 50$	$2 \begin{cases} 1: t = 8 \\ 1: \text{ reason} \end{cases}$
d. The absolute maximum velocity is 95 ft/sec at $t = 15$ $v'(t)$ decreases on the interval $0 < t < 4$, $v'(t)$ increases on the interval $4 < t < 15$ and $v'(t)$ decreases on the interval $15 < t < 20$ the candidates are $v(0) = 50$ ft/sec and $v(15) = v(0) + \int_0^{15} a(t)dt = 50 + 45 = 95$ ft/sec	3 { 1: $t = 15$ 1: absolute maximum velocity 1: identifies $t = 0$ and $t = 15$ as candidates or indicates that v decreases, increases and then decreases

2. For $-4 \le t \le 2$ the graph of a function f is shown below. Let $g(x) = \int_0^{\frac{1}{2}x} f(t)\,dt$.

The graph of $f(t)$



a. What is the domain of $g(x)$?

b. Compute, or state that it does not exist, $g(-2), g'(-2), g''(-2)$

c. Find all values of x where $g(x)$ has a relative minimum. Justify your answer.

d. Find all values of x in the open interval $(-8,4)$ at which the graph of g has a point of inflection.

---

a. $-4 \le \frac{1}{2}x \le 2$

$-8 \le x \le 4$

1: answer

b. $g(-2) = \int_0^{\frac{1}{2}(-2)} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt =$

$-\frac{1}{2}1(6+4) = -5$

$g'(x) = f(\frac{1}{2}x) \cdot \frac{1}{2}$

$g'(-2) = f(-1) \cdot \frac{1}{2} = (6) \cdot \frac{1}{2} = 3$

$g''(x) = f'(\frac{1}{2}x) \cdot \frac{1}{4}$

$g''(-2) = f'(-1) \cdot \frac{1}{4}$ Does not exist

5 $\begin{cases} 1: & g(-2) \\ 1: & g'(x) \\ 1: & g'(-2) \\ 1: & g''(x) \\ 1: & \text{answer} \end{cases}$

c. g has a relative minimum at

$\frac{1}{2}x = -3; x = -6$

This is the only x value where $g'(x)$ changes from negative to positive

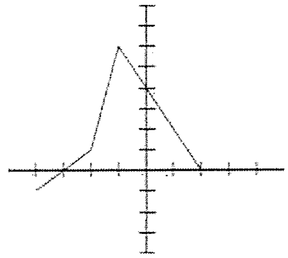
2 $\begin{cases} 1: & x = -6 \\ 1: & \text{justification} \end{cases}$

d. $x = -2$

1: answer

2. For $-4 \le t \le 2$ the graph of a function f is shown below. Let $g(x) = \int_0^{\frac{1}{2}x} f(t)\,dt$.

The graph of $f(t)$



a. What is the domain of $g(x)$?

b. Compute, or state that it does not exist, $g(-2), g'(-2), g''(-2)$

c. Find all values of x where $g(x)$ has a relative minimum. Justify your answer.

d. Find all values of x in the open interval $(-8,4)$ at which the graph of g has a point of inflection.

---

a. $-4 \le \frac{1}{2}x \le 2$

$-8 \le x \le 4$

1: answer

b. $g(-2) = \int_0^{\frac{1}{2}(-2)} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt =$

$-\frac{1}{2}1(6+4) = -5$

$g(-2)$ = area from -1 to 0 of that trapezoid $\frac{1}{2}(1)(4+6) = 5$ but bounds are flipped so -5

$g'(x) = f(x)$ but look @ bounds so

$g'(x) = f(\frac{1}{2}x)\cdot\frac{1}{2}$ by Fund Thm of CALC

$g'(-2) = f(-1)\cdot\frac{1}{2} = (6)\cdot\frac{1}{2} = 3$

$g''(x) = f'(\frac{1}{2}x)\cdot\frac{1}{4}$ ← Chain Rule

$g''(-2) = f'(-1)\cdot\frac{1}{4}$ Does not exist

"KINK" so derivative does not exist

5 { 1: $g(-2)$; 1: $g'(x)$; 1: $g'(-2)$; 1: $g''(x)$; 1: answer

c. g has a relative minimum at

$\frac{1}{2}x = -3; x = -6$

This is the only x value where $g'(x)$ changes from negative to positive

2 { 1: $x = -6$; 1: justification

GRAPH is $g'(x)$ so CRIT PTS @ -3 & 2 at $- - \; + + + \; ?$ (-3, 2) MIN

so $\frac{1}{2}x = -3$, $x = -6$

d. $x = -2$

Need to examine HORIZ TAN of $f(t)$ ONLY ONE @ -1

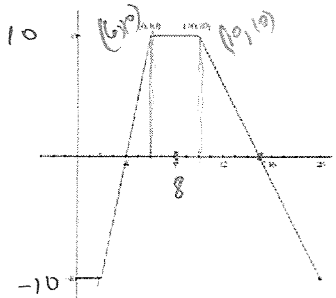
so $\frac{1}{2}x = -1$

1: answer

4. A car is traveling on a straight road with velocity 50 ft/sec at time $t = 0$. For $0 \le t \le 20$, the car's acceleration $a(t)$, in $ft/\sec^2$, is the piecewise linear function defined by the graph below.

Initial V = 50 ft/s

$a(t)$



a. Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

b. Is the speed of the car increasing or decreasing at time $t = 2$? Give a reason for your answer.

c. At what time in the interval $0 < t \le 20$ is the velocity of the car 50 ft/sec?

d. On the time interval $0 \le t \le 20$, what is the car's absolute maximum velocity, in ft/sec? At what time does it occur? Why?

---

a. Since $v'(2) = a(2) = -10$, the velocity is decreasing at $t = 2$

← AREA = (−)! UNDER GRAPH = v(t)

2 { 1: answer, 1: reason

$a(2) = -10$

b. $v(2) = v(0) + \int_0^2 a(t)\,dt = 50 + -20 = 30$ ft/sec

↑ Initial

Speed is decreasing since $a(2) < 0$ and $v(2) > 0$

If a(−) v(t) opp so decreasing!

2 { 1: decreasing, 1: reason

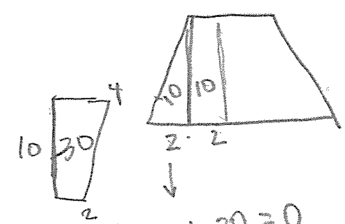
Initial Velocity 50 ←

c. at time $t = 8$

$v(8) = v(0) + \int_0^8 a(t)\,dt = 50 + 0 = 50$

Need Area to be 0

2 { 1: $t = 8$, 1: reason



$-30 + 10 + 20 = 0$

d. The absolute maximum velocity is 95 ft/sec at $t = 15$

$v'(t)$ decreases on the interval $0 < t < 4$,

$v'(t)$ increases on the interval $4 < t < 15$ and

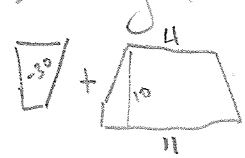
$v'(t)$ decreases on the interval $15 < t < 20$

the candidates are $v(0) = 50$ ft/sec

and $v(15) = v(0) + \int_0^{15} a(t)\,dt = 50 + 45 = 95$ ft/sec

3 { 1: $t = 15$
1: absolute maximum velocity
1: identifies $t = 0$ and $t = 15$ as candidates or indicates that v decreases, increases and then decreases

Need to Find max area of a(t) graph
Most likely candidate at t=15



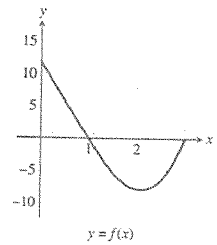
$= -30 + 75 = 45$ + Initial 50

$= 95$ ft/s

3. Let $F(x) = \int_0^x f(t)\,dt$, where $f(t)$ is the continuous function whose graph is shown.

(A) Where does F achieve its maximum value? Explain.

MAX @ 1 since MAX area ∴ (1,5) MAX

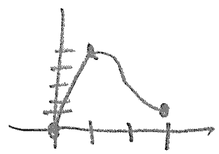


$y = f(x)$

(B) Where does F achieve its minimum value? Explain.

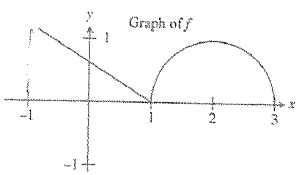
at 3 △ − ◡

(C) Sketch a possible graph for F on the interval [0, 3].



1. Evaluate $\frac{d}{dx}\int_x^5 2^{t^2}\,dt$. $-\int_5^x 2^{t^2}\,dt = -2^{x^2}$

2. Let f be the continuous function defined on $[-1, 3]$ whose graph, shown below, is comprised of a line segment and semicircle.



$g(-1) = \int_1^{-1} f(t)\,dt$

$= -\int_{-1}^{1} f(t)\,dt = -1$

$g(1) = 0$

$g(3) = \int_1^3 f(t)\,dt = \frac{1}{2}\pi$

If $g(x) = \int_1^x f(1)\,dt$, use the graph of f on the previous page to

(A) Find $g(-1)$, $g(1)$, and $g(3)$

(B) Find intervals on $[-1, 3]$ where g is decreasing. Never Area below (t

(C) Find intervals on $[-1, 3]$ where the graph of g would be concave downward. CRIT PT @ 1 & 3

Inflection @ 2

Concave down (2,3)

But CRIT pt @ 1 also so examine (−1,1)

Thus (−1,1) & (2,3)

3. Which of the following is the greatest value of x on the interval [0, 3] for which $\int_0^x (t^2 - 3t)\,dt \geq \int_2^x t\,dt$?

(A) 0.56 (B) 0.92 (C) 1.36 (D) 1.57 (E) 1.78

$\int_0^x = \frac{x^3}{3} - \frac{3x^2}{2}$

$\int_2^x = \frac{x^2}{2} - 2$

$\frac{x^3}{3} - \frac{3x^2}{2} \geq \frac{x^2}{2}$

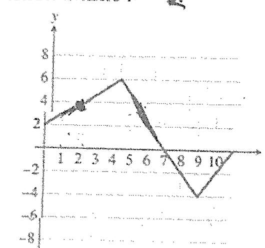
$2x^3 - 9x^2 \geq 3x^2$

$2x^3 - 6x^2 + 12z$

4. Let f be the differentiable function whose graph is shown. The position at time t (in seconds) of a particle moving along a coordinate axis is $s = \int_0^t f(x)\,dx$ meters.

(A) What is the particle's position at time $t = 2$?

$\frac{1}{2}(2)(6) = 6$



(B) At what time during the first 11 seconds does the particle's position have its largest value? Justify your answer.

At t = 7, Greatest Area