Additional Area Under Curve Ch 5 Examples for Test 1

1. The rate at which gas is flowing through a large pipeline is given in thousands of gallons per month in the chart below:

t (months)	0	3	6	9	12
R(t) (1000 gallons/month)	43	62	56	60	68

a) Use left Riemann sum to approximate the total gallons that flowed in the pipeline.

$\Delta x = 3$ $\quad 3[f(0)+f(3)+f(6)+f(9)]$

$3[43+62+56+60] \approx \boxed{663 \text{ thou gallons}}$

b) Use midpoints to approximate the total gallons that flowed in the pipeline.

$\Delta x\, f(\bar{x}_i)$ $\quad 3[f(1.5)+f(4.5)+f(7.5)+f(10.5)]$

$3\left[\frac{f(3)+f(0)}{2}+\frac{f(6)+f(3)}{2}+\frac{f(9)+f(6)}{2}+\frac{f(12)+f(9)}{2}\right] = 3[52.5+59+58+64]$

$= \boxed{700.5 \text{ th gall}}$

2. A 12 meter long tree trunk with circular cross sections of varying diameter are represented in the table below. The distance, x, of the tree trunk is measured from the ground and D(x) represents the diameter at that point.

x	0	2	4	6	8	10	12
D(x)	1.7	1.5	1.46	1.42	1.5	1.38	1.21

a) Write an integral expression in terms of D(x) that represents the volume of the tree trunk between x = 0 and x = 12.

Volume = Σ of all the circular areas on interval, thus $A = \pi r^2$ so $\boxed{V = \int_0^{12} \pi \left[\frac{D(x)}{2}\right]^2}$

b) Approximate the volume of the tree trunk between x = 0 and x = 12 using the data from the table and a midpoint Riemann sum with three subintervals of equal length.

Since 3 subintervals, $\Delta x = 4$ Thus need midpts between $f(0)$ & $f(4)$

$f(2) + f(6) + f(10)$

$\pi \cdot 4\left[\left(\frac{D(2)}{2}\right)^2 + \left(\frac{D(6)}{2}\right)^2 + \left(\frac{D(10)}{2}\right)^2\right]$

$\pi \cdot [1.5^2 + 1.42^2 + 1.38^2] = \boxed{19.4 \text{ m}}$

c) Explain why there must be a value x for 0 < x < 12 such that D'(x) = 0?

$D(2) = 1.5$
$D(8) = 1.5$ $\}$ continuous

Thus MVT guarantees at least 1 x in (2,8) such that $D'(x) = 0$!