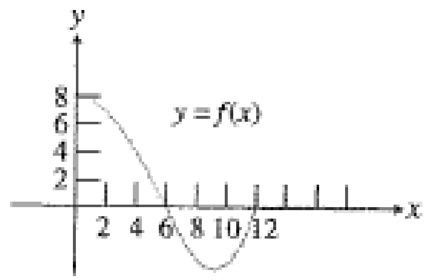
AP Calc – Yet another review for Chapter 5 – Test 1

1. Let $H(x) = \int_0^x f(t)dt$, where f is the continuous function with domain [0, 12] as shown in the graph.

a) Find H(0).
b) On what interval is H increasing? Explain.
c) On what interval is the graph of H concave up? Explain.
d) Is H(12) positive or negative? Explain.
e) Where does H achieve its maximum value? Explain.
f) Where does H achieve its minimum value? Explain.



2. The rate of fuel consumption (in gallons per minute) recorded during a plane flight is given by a twice differentiable function R of time t, minutes is shown in the table.

t	0	30	40	50	70	90
R(t)	20	30	40	55	65	70

a) Approximate the total fuel consumption using left Riemann Sums with 5 subintervals.

b) Is your approximation an over or underestimate? Explain why.

#3 Taken from:

2005 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

Answers:

1. a) 0 because $\int_0^0 = 0$ b) [0, 6] $\int_0^6 = \textit{positive}$ c) Need (+) slope, so (9, 12)

d) H(12) = $\int_0^{12} f(t)dt > 0$ will result in a (+) net area

e) H(6) since $\int_0^6$ will result in largest area

f) H(0) since area = 0. All other values will have a net area > 0

2. a) S = 20(30) + 30(10) + 40(10) + 55(20) + 65(20) = 3700 gallons
   b) Underestimate since the curve is increasing with left Riemann sums

3.

(a) $\dfrac{T(8)-T(6)}{8-6} = \dfrac{55-62}{2} = -\dfrac{7}{2}\,^\circ\text{C/cm}$	1 : answer
(b) $\dfrac{1}{8}\int_0^8 T(x)\,dx$ Trapezoidal approximation for $\int_0^8 T(x)\,dx$: $A = \dfrac{100+93}{2}\cdot 1 + \dfrac{93+70}{2}\cdot 4 + \dfrac{70+62}{2}\cdot 1 + \dfrac{62+55}{2}\cdot 2$ Average temperature $\approx \dfrac{1}{8}A = 75.6875^\circ\text{C}$	3 : { 1 : $\frac{1}{8}\int_0^8 T(x)\,dx$; 1 : trapezoidal sum; 1 : answer
(c) $\int_0^8 T'(x)\,dx = T(8) - T(0) = 55 - 100 = -45^\circ\text{C}$ The temperature drops 45°C from the heated end of the wire to the other end of the wire.	2 : { 1 : value; 1 : meaning
(d) Average rate of change of temperature on $[1, 5]$ is $\dfrac{70-93}{5-1} = -5.75$. Average rate of change of temperature on $[5, 6]$ is $\dfrac{62-70}{6-5} = -8$. No. By the MVT, $T'(c_1) = -5.75$ for some c_1 in the interval $(1, 5)$ and $T'(c_2) = -8$ for some c_2 in the interval $(5, 6)$. It follows that T' must decrease somewhere in the interval (c_1, c_2). Therefore T'' is not positive for every x in $[0, 8]$.	2 : { 1 : two slopes of secant lines; 1 : answer with explanation
Units of $^\circ\text{C/cm}$ in (a), and $^\circ\text{C}$ in (b) and (c)	1 : units in (a), (b), and (c)