KEY

AP Calculus – Chapter 5 – Integral and Area Under Curve Review

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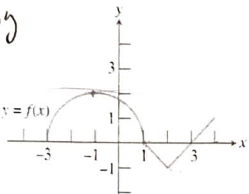
p.322-323

54. The graph of a function f consists of a semicircle and two line segments as shown below.



Let $g(x) = \int_1^x f(t)dt$.

a) Find g(1) $= 0$

b) Find g(3) $= -1$

$\int_1^3 = \frac{1}{2}\cdot 2\cdot(-1) = -1$

c) Find g(-1) $\int_1^{-1} = -\int_{-1}^{1} = -\frac{1}{2}(\text{semi-circle}) = -\frac{1}{2}\left(\frac{1}{2}\right)(\pi 2^2) = \boxed{-\pi}$

d) Find all values of x on the open interval (-3, 4) at which g has a relative ~~minimum~~ maximum.

$x = 1$ (+) y's on left of 1, (−) y's on right of 1

e) Write an equation for the tangent line to the graph of g at x = -1. $y = mx + b$ @ $x=-1$ $y = 2$

Since $g'(x) = f(t)$ then $m = 2$ @ $x = -1$, so $y = 2x + b$

Now Need area for coordinate so when $x = -1$ $y = -\pi$ from part b

$y = 2x + b$

$-\pi = 2(-1) + b$

$-\pi + 2 = b$

$\boxed{y = 2x - \pi + 2}$

f) Find the x-coordinate of each point of inflection of the graph of g on the open interval (-3, 4). ↳ Need $f'(t)$ to have a slope of 0 or undef

This occurs @ $x = -1$ & 2

AP *Examination Preparation

You may use a graphing calculator to solve the following problems.

58. The rate at which water flows out of a pipe is given by a differentiable function R of time t. The table below records the rate at 4-hour intervals for a 24-hour period.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
4	10.3
8	10.9
12	11.1
16	10.9
20	10.5
24	9.6

(a) Use the Trapezoidal Rule with 6 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Explain the meaning of your answer in terms of water flow, using correct units.

(b) Is there some time t between 0 and 24 such that $R'(t) = 0$? Justify your answer.

(c) Suppose the rate of water flow is approximated by $Q(t) = 0.01(950 + 25x - x^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour period. Indicate units of measure.

ⓐ $\int_0^{24} R(t)dt \approx \frac{4}{2}(9.6 + 2(10.3) + 2(10.9) + 2(11.1) + 2(10.9) + 2(10.5) + 9.6)$

$\approx \boxed{253.2}$

This is the total number of gal of water that flowed thru pipe in 24 hr period

ⓑ Yes by MVT $R(0) = R(24)$ so MVT guarantees a number c between 0 & 24 such that $R'(c) = 0$

ⓒ Ave Rate

$\frac{1}{24-0}\int_0^{24} Q(t)\,dt = 10.58$ gal/hr.

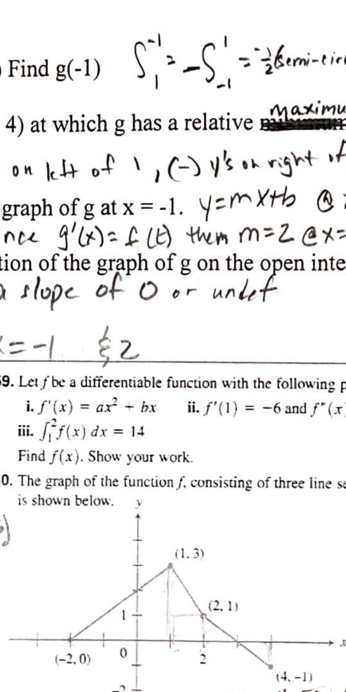
59. Let f be a differentiable function with the following properties.

i. $f'(x) = ax^2 + bx$ ii. $f'(1) = -6$ and $f''(x) = 6$

iii. $\int_1^2 f(x)\,dx = 14$

Find $f(x)$. Show your work.

60. The graph of the function f, consisting of three line segments, is shown below.



Let $g(x) = \int_1^x f(t)\,dt$.

(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 2$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. Which of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

ⓐ $\int_1^4 = \boxed{2}$ ⓑ $\int_1^{-2} = -\int_{-2}^{1} = \boxed{\frac{9}{2}}$

ⓑ By FTC $g'(2) = f(2) = \boxed{1}$

ⓒ $g'(x) = f(x)$ above axis, so (+) VALUES from (-2,3)
(−) Values from (3,4)

$g = \int_1^x f(t)$

Need to Compare $\int_1^4$ and $\int_{-2}^{1}$ thus $g(-2) = \boxed{\frac{9}{2}}$

ⓓ @ $x = 2$, slope stays the same (−)