AP Calculus
2017-18 Semester 1 Most Missed

Name: KEY

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1. If $f''(x) = (x-3)(x+4)^2(x-2)$, find all inflection points.

Possible $x = 3, -4, 2$ → test

+++ | +++ | - - - | +++
$-4 \quad 2 \quad 3$

ONLY @ $\boxed{x = 2 \,\&\, 3}$

2. $\lim_{h\to 0} \frac{\cos\left(\frac{\pi}{2}+h\right)-\cos\frac{\pi}{2}}{h} =$ $\quad -\sin\frac{\pi}{2} = \boxed{-1}$

deriv of cos

3. From Limit Laws in Section 2.2

* a) $\lim_{x\to 0} \frac{\sin^2 3x}{3x} = \lim_{x\to 0} \sin 3x \cdot \frac{\sin 3x}{3x}$ b) $\lim_{x\to 0} \frac{1-\cos x}{x}$ $= \boxed{0}$

$\lim_{x\to 0} \sin 3x \cdot \lim_{x\to 0} \frac{\sin 3x}{3x}$

$= 0 \cdot 1 = \boxed{0}$

4. If $f'(x) = \cos x$ and $f(\pi/2) = 4$, then $f(x) = ?$

$\boxed{\sin x + 3} = f(x)$

$\int \cos x\, dx = \sin x + C$

$\sin\frac{\pi}{2} + C = 4$

$1 + C = 4$ $\quad C = 3$

5. Use the table to find the following:

$\frac{d}{dx}(f(g(3)))$

x	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
$g(x)$	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
$f'(x)$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
$g'(x)$	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$

$\frac{d}{dx} f(g(x))$

$f'(g(x)) \cdot g'(x)$

$f'(g(3)) \cdot g'(3) \Rightarrow f'(2) \cdot (-3)$

$\frac{1}{4} \cdot (-3) = \boxed{\frac{-3}{4}}$

6. If $g(x) = \frac{-3x - f(x)}{f(x)}$ and $f(1) = 2$ and $f'(1) = -3$, then $g'(1) = ?$

$\frac{f(x)\cdot(-3 - f'(x)) - (-3x - f(x))(f'(x))}{[f(x)]^2} = \frac{2(-3-(-3)) - (-3\cdot1-2)(-3)}{4}$

$= \frac{2(0) - (-5)(-3)}{4} = \boxed{\frac{-15}{4}}$

7. $\int_{-3}^{1} \sqrt{5} x^{-3}$

$\frac{\sqrt{5}}{-2} x^{-2} \Big|_{-3}^{1}$

$\frac{\sqrt{5}}{-2} - \frac{\sqrt{5}}{-2}\left(\frac{1}{(-3)^2}\right) = \frac{\sqrt{5}}{-2} - \frac{\sqrt{5}}{-18}$

$= \frac{-9\sqrt{5} + \sqrt{5}}{18} = \frac{-8\sqrt{5}}{18} = \boxed{\frac{-4\sqrt{5}}{9}}$

8. If a) $f(x) = 5$, then $f'(2) = ?$ $\quad \boxed{0}$

b) $f(x) = \pi^5$, then $f'(2) = ?$ $\quad \boxed{0}$

9. What is the slope of the line tangent to y = cos²(2x + π) at x = 3π/4 ?

$y' = 2\cos(2x+\pi) \cdot -\sin(2x+\pi) \cdot 2$

@ $\frac{3\pi}{4} \rightsquigarrow 2\cos\left(2\frac{3\pi}{4}+\pi\right) \cdot -\sin\left(2\cdot\frac{3\pi}{4}+\pi\right)\cdot 2$

$= 2\cos\left(\frac{3\pi}{2}+\pi\right)\cdot -\sin\left(\frac{3\pi}{2}+\pi\right)\cdot 2$

$= 0$

or ON CALC w/ nDeriv $= \boxed{0}$

10. Find the horizontal asymptotes of $f(x) = \frac{2-|x|}{x}$

Need to Consider

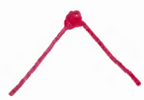
$\lim_{x\to -\infty} f(x) = \frac{2-(-x)}{x} = +1$

$\lim_{x\to +\infty} f(x) = \frac{2-(x)}{x} = -1$

$\boxed{y = \pm 1}$

11. If *f(c)* is a local maximum of a continuous function *f* on an open interval (a, b), then *f′(c) = 0.* Is this true or false? Justify your answer.

FALSE



← can be a kink, thus f′(c) = undef !!

12. Find all critical values and relative max/min of each:

a) $f'(x) = \frac{2}{3}x^{-1/3}$ $\rightsquigarrow$ $\int \frac{2}{3}x^{-\frac{1}{3}} = x^{\frac{2}{3}} + C$

$\frac{2}{3}x^{-\frac{1}{3}} = 0$

$\frac{2}{3\sqrt[3]{x}} = 0$

$- - - \;|\; + + +$ at 0

Min @ x = 0 verified

b) $f'(x) = \frac{x^2-5}{x} = 0$ $\rightsquigarrow$ $\int x - \frac{5}{x}$

$\frac{1}{2}x^2 - 5\ln|x| + C$

$x = \pm\sqrt{5}$ & ~~undef~~ 0

TEST

$- - \; + \; - - \; + + +$ at $-\sqrt{5}$, 0, $\sqrt{5}$

But this is a discontinuity @ x = 0.

ONLY MIN @ $x = \pm\sqrt{5}$

13. A particle moves along the x-axis and the position for 0 ≤ t ≤ 10 is given by $s(t) = 3\cos(\frac{\pi}{3}t)$
Find the acceleration of the particle at t = 4. Is the particle speeding up, slowing down or neither?

at t = 4?

$s'(t) = -3\sin\left(\frac{\pi}{3}t\right)\cdot\frac{\pi}{3}$

$s'(t) = -\pi\sin\left(\frac{\pi}{3}t\right)$

$s''(t) = -\pi\cos\left(\frac{\pi}{3}t\right)\cdot\frac{\pi}{3}$

$s''(t) = \frac{-\pi^2}{3}\cos\left(\frac{\pi}{3}t\right)$

$s''(4) = \frac{-\pi}{4}\cos\left(\frac{4\pi}{3}\right) \nearrow \frac{-\pi}{4}\left(\frac{-1}{2}\right) = \boxed{\frac{\pi}{8}}$

v: $- - - \;|\; - - - - \;|\; + + + +$ at 0, 3

a: $- - - \;|\; + + + \;|\; + + + \;|\; +$ at 0, 1.5, 4, 4.5

Speeding up