Semester 1 Review Answers

Free Response CALC AND NO CALC
***NOTE: SKIP #3 on CALC PORTION*
SKIP #5C on NO CALC PORTION, You do not know how to do this yet!

① $v(t) = 3t^2 - 4t - 4$

ⓐ $x(t) = \int v(t) = t^3 - 2t^2 - 4t + C$
when $t = 1$ $x(t) = 1$
so $1^3 - 2(1^2) - 4(1) + C = 1$
$1 - 2 - 4 + C = 1$
$C = 6$
$\boxed{x(t) = t^3 - 2t^2 - 4t + 6}$

ⓑ $v(t) = 3t^2 - 4t - 4$ INST
$V_{ave} = \frac{x(4) - x(0)}{4 - 0} = \frac{22 - 6}{4} = 4$
$x(4) = 4^3 - 2(4^2) - 4(4) + 6 = 22$
$x(0) = 6$
Thus $3t^2 - 4t - 4 = 4$
$3t^2 - 4t - 8 = 0$
$\boxed{t \approx 2.431}$ & $\cancel{-1.097}$

ⓒ $v(t) = 3t^2 - 4t - 4$
$0 = 3t^2 - 4t - 4$
$0 = (3t + 2)(t - 2)$
$v(t) = 0$ @ $t = 2$ & $t = \frac{-2}{3}$
--- 2 +++
moving backward moving forward

So need dist from $[0, 2] \cup [2, 4]$
$|x(2) - x(0)| + |x(4) - x(2)| = |-2 - 6| + |22 - -2|$
$8 + 24 =$
$\boxed{32}$

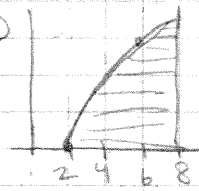
② $M(\frac{\pi}{2}, 4)$
$N(\pi, 0)$

ⓐ $m = \frac{0 - 4}{\pi - \frac{\pi}{2}} = \frac{-4}{\frac{\pi}{2}} = \frac{-8}{\pi}$
$y - 0 = \frac{-8}{\pi}(x - \pi)$
$\boxed{y = \frac{-8}{\pi}(x - \pi)}$

ⓑ $f(x) = 4\sin x$
$f'(x) = 4\cos x$ @ $(\pi, 0)$
$f'(\pi) = 4\cos\pi = m = -4$
$y - 0 = -4(x - \pi)$
$\boxed{y = -4(x - \pi)}$

ⓒ Need slope $= \frac{-8}{\pi}$
Thus $4\cos x = \frac{-8}{\pi}$
$\cos x = \frac{-2}{\pi}$
$\boxed{x = 2.26}$

④ ⓐ



ⓑ $A = \int_2^8 3\sqrt{x-2}\,dx = \int_2^8 3(x-2)^{\frac{1}{2}}\,dx$
$= 2(x-2)^{\frac{3}{2}}\Big]_2^8 = 2(6)^{\frac{3}{2}} - 0 = 2\sqrt{6^3} = \boxed{12\sqrt{6}}$

ⓒ $\boxed{A(k) = \int_2^k 3\sqrt{x-2}\,dx}$

ⓓ Need $\frac{dA}{dk} \Rightarrow \frac{dA}{dk} = \frac{d}{dk}\int_2^k 3\sqrt{x-2}\,dx = 3\sqrt{k-2}$ ← Fundamental Thm of CALC
When $k = 8$ $\Rightarrow 3\sqrt{8-2}$
$= \boxed{3\sqrt{6}}$

(5) (a) $f'(x) = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

0 & 2 crit pts

$f''(x) = 6x - 6$

$f''(0) = -$ ⌒ $\boxed{\text{MAX @ } (0, K)}$

$f''(2) = +$ ∪ $\boxed{\text{MIN @ } (2, -4+K)}$

(b) $f(x)$ = CUBIC ∴ all 3 ROOTS REAL!

∴ all values of K

(c) SKIP THEY Do NOT KNOW AVG VALUE

(6) (a) $g(5) \to g(x) = \int_0^x f(t)\,dt$

$\frac{1}{2}(3)(3) + -\frac{1}{2}\left(\frac{1}{2}\right)\pi 2^2$

$\boxed{\frac{9}{2} - \pi}$

(b) $g(x) = \int_0^x f(t)\,dt$

Thus $g'(x) = f(t)$ by Fund Thm of Calc

So crit pts are $-3, 3, 7$ (zero's)

But only 3 is viable

@ 3 +++ | − − − MAX

3

∴ Rel Max @ $x = 3$

(c) at $x = 5$ $m = -2$

and $g(5) = \frac{9}{2} - \pi \rightsquigarrow \left(5, \frac{9}{2} - \pi\right)$

so

$\boxed{y - \left(\frac{9}{2} - \pi\right) = -2(x - 5)}$

(d) $[-3, 7] \to$ Inflection pts possible at 0, 3, & 5

slope

+++ | − − | − − | +++

−3 0 3 5 7

Inflect where changes concavity, so at $\boxed{x = 0 \text{ \& } x = 5}$

Multiple Choice
Calc OK

(44) $f''(x) < 0$ ∴ concave down

∴ ~~f(x)~~ must be

3 4

Also $\frac{\Delta f(x)}{\Delta x}$ $\frac{2.68 - 2.48}{3.3 - 3.2} = \frac{.2}{.1} = 2$

$\frac{2.86 - 2.68}{3.4 - 3.3} = \frac{.18}{.1} = 1.8$

Thus $\boxed{1.8 < f'(3.3) < 2}$

(30) $y' = x^2 - 2x - 5 + 3\cos x$

$y'' = 2x - 2 - 3\sin x = 0$

GRAPH IT

FIND X-INT = 2.21

(B)

(31) graph of f shown and $\int_1^4 f(x)\,dx = 3.8$

Thus area under graph from 1 to 4 = 3.8

Since $F'(x) = f(x)$

Then $\int F'(x) = \int f(x)$

Need $\underline{0}$ to 4 so

0 to 1 + 1 to 4

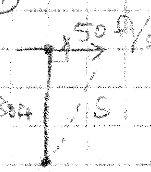
3 + 3.8 = (6.8) (D)

(32) (C) $\lim_{h\to 0} \frac{f(7+h) - f(7)}{h} = 4$

Limit exists so f is cont & differentiable

Cannot verify III so only 1 & 2

(34)

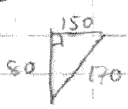


$x^2 + y^2 = s^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$ ($\frac{dy}{dt} = 0$)

$2(150)(50) = 2(170)\frac{ds}{dt}$

@ 3 seconds



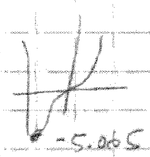
$\boxed{44.12 = \frac{ds}{dt}}$

(B)

(35) $y = 3x + 6$ MIN VALUE of x^3y

$x^3(3x+6)$ ← minimum

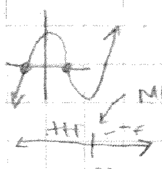
GRAPH IT



(B)

(38) GRAPH IT Need zeros of $f'(x)$ = CRIT PTS

zero's @ −.494 & 1.091



(C)

(39) $f(x) = \sqrt{2x}$

$f'(x)$ = RATE OF CHANGE

$f'(x) = \frac{1}{2}(2x)^{-\frac{1}{2}} \cdot 2 = \frac{1}{\sqrt{2x}}$

at $x = c$ $4 \cdot x = 1$

$f'(c) = 4f'(1)$

$\frac{1}{\sqrt{2c}} = \frac{4}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{4} = \sqrt{2c}$

$\frac{2}{16} = 2c = \boxed{\frac{1}{16}}$

(A)

(40) Since $a(t) = t + 2\sin t$

$\int a(t) = v(t)$

$v(t) = \frac{1}{2}t^2 - 2\cos t + C$

@ t=0 $-4 = \frac{1}{2}(0)^2 - 2\cos(0) + C$

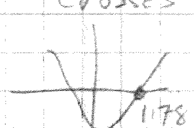
$-4 = -2 + C$

$-2 = C$

$v(t) = \frac{1}{2}t^2 - 2\cos t - 2 = 0$

GRAPH IT, FIND Where Crosses

(C)



(41) D (Think areas & zeros horiz tan at a & b)

(42) $\frac{1}{2}(\overset{\Delta x}{.5})\left[8 + 2(5) + 2(4) + 2(3) + 2(3) + 2(5) + 8\right]$

= 14 (B)

(43) I $(\sin 2x)(2)$ = $4\sin x\cos x$ (double ∠ formula)

II $4\sin x\cos x$

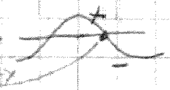
III $-4\cos x(-\sin x)$

(E) I, II, III

(45) $\frac{d}{dx}\int_0^x -\sin t^2\,dt = 0$ yields CRIT PTS

So GRAPH $y = -\sin x^2$

CRIT PTS = 1.77, 2.507 ← MAX



(E)

NO CALC

① $x^3-2x^2\Big]_1^3=(27-18)-(1-2)=9+1=10$

(C)

② $f'(x)=x\left[\frac{1}{2}(4x-1)^{-\frac{1}{2}}\cdot 4\right]+\sqrt{4x-1}$

$=\frac{2x}{\sqrt{4x-1}}+\frac{\sqrt{4x-1}}{1}=\frac{2x+4x-1}{\sqrt{4x-1}}$

$=\frac{6x-1}{\sqrt{4x-1}}$ (A)

③ $4a+b+\int_a^b 7\,dx=4a+b+7x\Big]_a^b$

$=4a+b+(7b-7a)=\boxed{-3a+8b}$

(C)

④ $f'(x)=-5x^4+1+-2x^{-3}$ @ $f'(-1)$

$=-5+1+\frac{-2}{-1}=-5+3=\boxed{-2}$

(C)

⑤ $y'=20x^3-72x^2+48x$

$y''=60x^2-144x+48=0$

$12(5x^2-12x+4)=0$

$12(5x-2)(x-2)$ +++ --- +++ $\frac{2}{5}$ 2

(E) $\frac{2}{5}<x<2$

⑫ $4x-5=8y$ need $m=\frac{1}{2}$ $y'=x=\frac{1}{2}$

$\frac{1}{2}x-\frac{5}{8}=y$ Thus $\left(\frac{1}{2},\frac{-11}{8}\right)$ $\frac{1}{8}-\frac{3}{2}$ $\frac{1}{8}-\frac{12}{8}$

(B)

⑬ need $\frac{\sqrt{9-x^2}}{x-3}<0$ when $x<3$ thus $(-\infty,3)$

~~19-x²<0~~

(A)

⑭ $f'(5)=2$ $L(x)=f(a)+f'(a)(x-a)$

$3+2(x-5)$

$3+2x-10$

$2x-7$

Need $L(x)=0=2x-7$

$7=2x$

(C) $\boxed{3.5}=\frac{7}{2}=x$

⑮ B

⑰ $2x=-2y\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{x}{-y}$

$\frac{d^2y}{dx^2}=\frac{-y-x(-1)\frac{dy}{dx}}{y^2}$ $\frac{-y+x\left(\frac{x}{-y}\right)}{y^2}$ @ $(3,4)$ (A)

$\frac{-\left(4+\frac{9}{4}\right)}{16}=\frac{\frac{-16+9}{4}}{16}=\frac{-25}{64}$

(7) $\frac{d}{dx}\cos^3 x^2 = \frac{d}{dx}(\cos x^2)^3$

$= 3(\cos x^2)^2 \cdot -\sin x^2 \cdot 2x$

$= -6x\cos^2 x^2 \sin x^2$ (E)

(8) @ 4 (B) ← turns (−)

(9) Find Area $|\ | + |\ |$

$\left|\frac{1}{2}(4)(3)\right| + \left|\frac{1}{2}(-1)(4+2)\right|$

$|6| + |-3| = \boxed{9}$ (C)

TOTAL DIST = |TOTAL AREA|

(10) $y = (-\sin 3x)3$ @ $\frac{\pi}{6}$ $y = \cos 3\left(\frac{\pi}{6}\right)$

$m = (-\sin\frac{\pi}{2})3 = -3\sin\frac{\pi}{2}$ $\left(\frac{\pi}{6}, 0\right)$

$y - 0 = -3\sin\frac{\pi}{2}\left(x - \frac{\pi}{6}\right)$

$y = 3\sin\frac{\pi}{2}\left(x - \frac{\pi}{6}\right) \Rightarrow y = -3\left(x - \frac{\pi}{2}\right)$

(C)

(11) D

(24) 8, $\Delta x = \frac{1}{30}$ $f(x) = \sin x$ from 0 to 1

$\frac{1}{30} \cdot 30 = 1$

$\int_0^1 \sin x\,dx$ (B)

(26) Def $\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

Thus $\frac{(x+h)^2 - x^2}{h} \Rightarrow x^2$

So $2^2 = 4$

(D)

(27) $\lim_{x\to 1^-} -2$ $\lim_{x\to 1^+} 8$ $\neq$

$f'(x) = \begin{matrix} 6x & x \le 1 \\ 6 & x > 1 \end{matrix}$ YES, exists

(B)

(28) $\frac{d}{dx}\int_0^{x} \sqrt{t^2+9}\,dt = \sqrt{x^2+9} = \sqrt{4^2+9} = \boxed{5}$ (E)

Calculus AB—Exam 2: Section II, Part A

1. (A) $x(t) = t^3 - 2t^2 - 4t + c$

$x(1) = 1^3 - 2(1)^1 - 4(1) + c = 1 \Rightarrow c = 6$

$x(t) = t^3 - 2t^2 - 4t + 6$

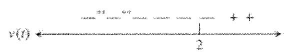
(B) $$v_{\text{avg}} = \frac{x(4) - x(0)}{4} = \frac{22 - 6}{4} = 4$$

$v(t) = 3t^2 - 4t - 4 = 4$

$3t^2 - 4t - 8 = 0$

$$t = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)} = \frac{4 \pm \sqrt{112}}{6} = \frac{4 + \sqrt{112}}{6} \approx 2.431$$

(C) $v(t) = 3t^2 - 4t - 4 = 0 \Rightarrow (3t + 2)(t - 2) = 0 \Leftrightarrow t = 2$



Change of direction at $t = 2$ since v changes sign there.

$x(0) = 6$ and $x(2) = -2$ and $x(4) = 22$

Total distance $= |x(2) - x(0)| + |x(4) - x(2)| =$

$|-2 - 6| + |22 - (-2)| = 8 + 24 = 32$

2. (A) $\text{Slope} = \dfrac{0 - 4}{\pi - \frac{\pi}{2}} = \dfrac{-4}{\frac{\pi}{2}} = \dfrac{-8}{\pi}$ Point $= (\pi, 0)$

$y - 0 = \dfrac{-8}{\pi}(x - \pi)$ or $y = \dfrac{-8}{\pi}x + 8$

(B) $f'(x) = 4\cos x$

Slope $= f'(\pi) = 4\cos\pi = -4$ Point $= (\pi, 0)$

$y = -4(x - \pi)$ or $y = -4x + 4\pi$

(C) $4\cos x = \dfrac{-8}{\pi} \Rightarrow x \approx 2.261$

(D) Using the washer method: $\pi \int_{\pi/2}^{\pi} \left[(4\sin x)^2 - \left(-\frac{8}{\pi}x + 8\right)^2 \right] dx$

3. (A) $\dfrac{dv}{dt} = -3(v + 5) \Rightarrow \dfrac{dv}{v + 5} = -3\,dt \Rightarrow \displaystyle\int \frac{dv}{v + 5} = \int -3\,dt$

$\ln|v + 5| = -3t + c_1 \Rightarrow |v + 5| = e^{-3t + c_1} \Rightarrow |v + 5| = e^{-3t}e^{c_1}$

$\Rightarrow v + 5 = ce^{-3t} \Rightarrow v = ce^{-3t} - 5$

$-15 = ce^{-3(0)} - 5 \Rightarrow -15 = c - 5 \Rightarrow c = -10 \Rightarrow v = -10e^{-3t} - 5$

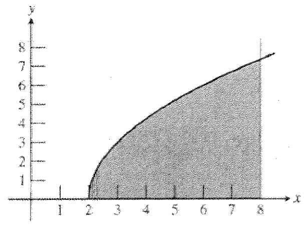
(B) $\lim_{t\to\infty} (-10e^{-3t} - 5) = -10(0) - 5 \Rightarrow -5$ m/sec

(C) $-10e^{-3t} - 5 = -5.3 \Rightarrow -10e^{-3t} = -0.3 \Rightarrow e^{-3t} = 0.03 \Rightarrow t = \dfrac{\ln 0.03}{-3} \approx 1.169$

The crate will be able to land safely after about 1.169 seconds.

Calculus AB—Exam 2: Section II, Part B

4. (A)



(B) $\int_2^8 3(x-2)^{1/2}\,dx = \left[3\left(\frac{2}{3}\right)(x-2)^{3/2}\right]_2^8$

$= \left[2(x-2)^{3/2}\right]_2^8 = 2(6)^{3/2} - 2(0)^{3/2}$

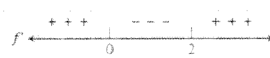
$= 12\sqrt{6}$

(C) $A(k) = \int_2^k 3\sqrt{x-2}\,dx$

(D) $\frac{dA}{dk} = \frac{d}{dk}\left(\int_2^k 3\sqrt{x-2}\,dx\right) = 3\sqrt{k-2}$ (Fundamental Theorem of Calculus)

When $k = 8$, $\frac{dA}{dk} = 3\sqrt{8-2} = 3\sqrt{6}$.

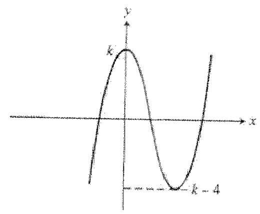
5. (A) $f'(x) = 3x^2 - 6x = 3x(x-2)$



There is a relative maximum at $x = 0$ because the function changes from increasing ($f'(x) > 0$) to decreasing ($f'(x) < 0$) there. There is a relative minimum at $x = 2$ because the function changes from decreasing ($f'(x) < 0$) to increasing ($f'(x) > 0$) there.

$f(0) = k$ is a relative maximum and $f(2) = k - 4$ is a relative minimum.

(B) In a cubic equation, we will only have three distinct roots if the relative maximum lies above the x-axis and the relative minimum lies below the x-axis (see the figure).



$$k > 0 \quad \text{and} \quad k - 4 < 0$$
$$0 < k < 4$$

(C) $$f_{\text{avg}} = \frac{1}{1-(-2)}\int_{-2}^{1}(x^3 - 3x^2 + k)\,dx = \frac{1}{3}\int_{-2}^{1}(x^3 - 3x^2 + k)\,dx$$

$$= \frac{1}{3}\left[\frac{x^4}{4} - x^3 + kx\right]_{-2}^{1} = \frac{1}{3}\left[\left(\frac{1}{4} - 1 + k\right) - (4 + 8 - 2k)\right]$$

$$= \frac{1}{3}\left(3k - \frac{3}{4} - 12\right) = k - 4.25$$

$k - 4.25 = 2 \Rightarrow k = 6.25$

6. (A) $g(5) = \frac{1}{2}(3)(3) - \frac{1}{4}\pi(2)^2 = \frac{9}{2} - \pi$

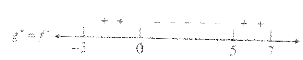
(B) By the Fundamental Theorem of Calculus, we know that $g'(x) = f(x)$, the function whose graph is depicted. For the interval given, $g(x)$ has no endpoints and only one critical point, $x = 3$. Since $g'(x) = f(x)$ changes from positive to negative at that point, we know that $g(x)$ has a relative maximum at that point.

(C) $g(5) = \frac{9}{2} - \pi \Rightarrow$ Point: $\left(5, \frac{9}{2} - \pi\right)$

From the graph, $f(5) = g'(5) = -2 \Rightarrow$ Slope $= -2$

Tangent line: $y - \left(\frac{9}{2} - \pi\right) = -2(x - 5)$ or $y = -2x + \frac{29}{2} - \pi$

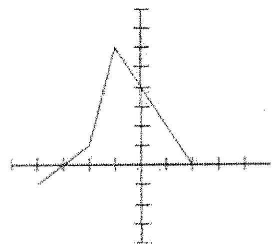
(D) Since g is differentiable for the interval $(-3, 7)$, g is continuous on $[-3, 7]$.



g has points of inflection at $x = 0$ and at $x = 5$ since g'' changes signs at both of these points.

2. For $-4 \le t \le 2$ the graph of a function f is shown below. Let $g(x) = \int_0^{\frac{1}{2}x} f(t)\, dt$.

The graph of $f(t)$



a. What is the domain of $g(x)$?
b. Compute, or state that it does not exist, $g(-2), g'(-2), g''(-2)$
c. Find all values of x where $g(x)$ has a relative minimum. Justify your answer.
d. Find all values of x in the open interval $(-8,4)$ at which the graph of g has a point of inflection.

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a. $-4 \le \frac{1}{2}x \le 2$

$-8 \le x \le 4$

1: answer

b. $g(-2) = \int_0^{\frac{1}{2}(-2)} f(t)\, dt = -\int_{-1}^{0} f(t)\, dt =$

$-\frac{1}{2}1(6+4) = -5$

$g(-2)$ = area from -1 to 0 of that trapezoid $\frac{1}{2}(1)(4+6) = 5$ but bounds are flipped so -5

$g'(x) = f(x)$ but look @ bounds so

$g'(x) = f(\frac{1}{2}x)\cdot\frac{1}{2}$ by Fund Thm of CALC

$g'(-2) = f(-1)\cdot\frac{1}{2} = (6)\cdot\frac{1}{2} = 3$

$g''(x) = f'(\frac{1}{2}x)\cdot\frac{1}{4}$ ← Chain Rule

$g''(-2) = f'(-1)\cdot\frac{1}{4}$ Does not exist

"KINK" so derivative does not exist

5: 1: $g(-2)$; 1: $g'(x)$; 1: $g'(-2)$; 1: $g''(x)$; 1: answer

c. g has a relative minimum at

$\frac{1}{2}x = -3; x = -6$

This is the only x value where $g'(x)$ changes from negative to positive

2: 1: $x = -6$; 1: justification

GRAPH is $g'(x)$ so CRIT PTS @ -3 & 2 at -3, 2 MIN ? so $\frac{1}{2}x = -3$ $x = -6$

d. $x = -2$

Need to examine HORIZ TAN of $f(t)$ ONLY ONE @ -1

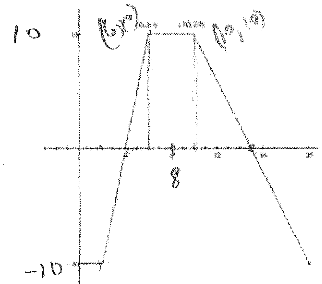
1: answer

so $\frac{1}{2}x = -1$ $x = -2$

4. A car is traveling on a straight road with velocity 50 ft/sec at time $t = 0$. For $0 \le t \le 20$, the car's acceleration $a(t)$, in $ft/\sec^2$, is the piecewise linear function defined by the graph below.

Initial $V = 50$ ft/s

$a(t)$



a. Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

b. Is the speed of the car increasing or decreasing at time $t = 2$? Give a reason for your answer.

c. At what time in the interval $0 < t \le 20$ is the velocity of the car 50 ft/sec?

d. On the time interval $0 \le t \le 20$, what is the car's absolute maximum velocity, in ft/sec? At what time does it occur? Why?

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a. Since $v'(2) = a(2) = -10$, the velocity is decreasing at $t = 2$

AREA = (-)! UNDER GRAPH - v(t)

2 { 1: answer; 1: reason

$a(2) = -10$

b. $v(2) = v(0) + \int_0^2 a(t)\,dt = 50 + -20 = 30$ ft/sec

Initial

Speed is decreasing since $a(2) < 0$ and $v(2) > 0$

If a(-) v(t) opp so decreasing!

2 { 1: decreasing; 1: reason

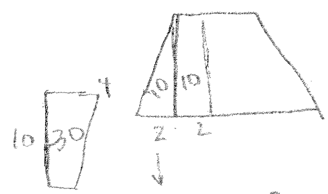
Initial Velocity 50ft

c. at time $t = 8$

$v(8) = v(0) + \int_0^8 a(t)\,dt = 50 + 0 = 50$

Need Area to be 0

2 { 1: $t = 8$; 1: reason



seconds 4+2+2 = 8

$-30 + 10 + 20 = 0$

d. The absolute maximum velocity is 95 ft/sec at $t = 15$

$v'(t)$ decreases on the interval $0 < t < 4$,

$v'(t)$ increases on the interval $4 < t < 15$ and

$v'(t)$ decreases on the interval $15 < t < 20$

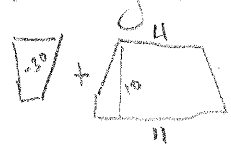
the candidates are $v(0) = 50$ ft/sec

and $v(15) = v(0) + \int_0^{15} a(t)\,dt = 50 + 45 = 95$ ft/sec

3 { 1: $t = 15$; 1: absolute maximum velocity; 1: identifies $t = 0$ and $t = 15$ as candidates or indicates that v decreases, increases and then decreases

Need to Find max area of a(t) graph

Most likely candidate at t=15

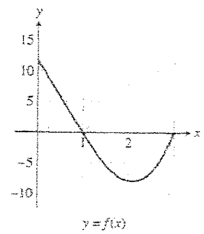


$= -30 + 75 = 45 +$ Initial 50

$= 95$ ft/s

3. Let $F(x) = \int_0^x f(t)\,dt$, where $f(t)$ is the continuous function whose graph is shown.

(A) Where does F achieve its maximum value? Explain.



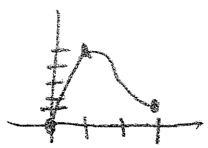
MAX @ 1 since MAX area

∴ (1,5) MAX

(B) Where does F achieve its minimum value? Explain.

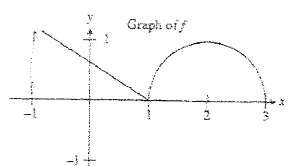
$\boxed{\text{at } 3}$ △ - ▽

(C) Sketch a possible graph for F on the interval $[0, 3]$.



1. Evaluate $\frac{d}{dx}\int_x^5 2^{t^2}\,dt$. $\quad -\int_5^x 2^{t^2}\,dt \quad = \boxed{-2^{x^2}}$

2. Let f be the continuous function defined on $[-1, 3]$ whose graph, shown below, is comprised of a line segment and semicircle.



$g(-1) = \int_1^{-1} f(t)\,dt$

$= -\int_{-1}^{1} f(t)\,dt = \boxed{-1}$

$g(1) = 0$

$g(3) = \int_1^3 f(t)\,dt = \boxed{\frac{1}{2}\pi}$

If $g(x) = \int_1^x f(t)\,dt$, use the graph of f on the previous page to

(A) Find $g(-1)$, $g(1)$, and $g(3)$

(B) Find intervals on $[-1, 3]$ where g is decreasing. Never Area below (t)

(C) Find intervals on $[-1, 3]$ where the graph of g would be concave downward. CRIT PT @ 1 & 3

Inflection @ 2

+++ | - - -
2

Concave down (2,3)

But CRIT pt @ -1 also So examine (-1,1)

Thus (-1,1) & (2,3)

Which of the following is the greatest value of x on the interval $[0, 3]$ for which $\int_0^x (t^2 - 3t)\,dt \geq \int_2^x t\,dt$?

(A) 0.56 (B) 0.92 (C) 1.36 (D) 1.57 (E) 1.78

+++ +++ | - - -
$\frac{3}{4}$ 3

$\int_0^x = \frac{x^3}{3} - \frac{3x^2}{2}$

$\int_2^x = \frac{x^2}{2} - 2$

$\frac{x^3}{3} - \frac{3x^2}{2} \geq \frac{x^2}{2}$

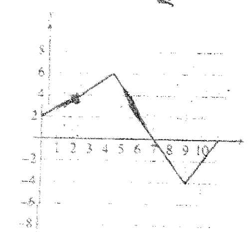
$2x^3 - 9x^2 \geq 3x^2$

$2x^3 - 6x^2 + 12 \geq$

Let f be the differentiable function whose graph is shown. The position at time t (in seconds) of a particle moving along a coordinate axis is $s = \int_0^t f(x)\,dx$ meters.

(A) What is the particle's position at time $t = 2$?

$\frac{1}{2}(2)(6) = \boxed{6}$



(B) At what time during the first 11 seconds does the particle's position have its largest value? Justify your answer.

At t = 7, Greatest Area

KEY

AP Calculus – Additional Semester 1 Review Problems

While driving to work, Mr. Matlack's velocity was $v(t) = 15t + 10$, where t is hours and $v(t)$ is miles per hour. Determine how far Mr. Matlack lives from school if it takes him:

a. 1 hour to get to work. $10 + \frac{1}{2}(1)(15)$ $\boxed{17.5}$

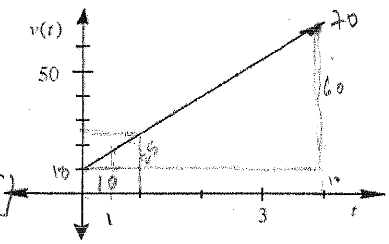
b. 4 hours to get to work. $4(10) + \frac{1}{2}(4)(60) = \boxed{160}$

c. $\frac{1}{2}$ hour to get to work. $\frac{1}{2}(10) + \frac{1}{2}(\frac{1}{2})(15(\frac{1}{2})+10-10) = \boxed{6.875}$

d. t hours to get to work.

$10t + \frac{1}{2}(t)(15t)$

$10t + \frac{15}{2}t^2$ or $\int v(t) = \boxed{\frac{15}{2}t^2 + 10t}$



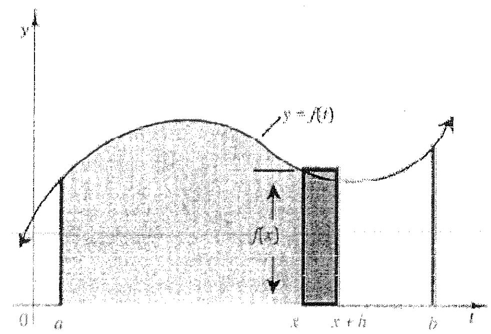
Now examine $A'(x)$ analytically.

If $A(x) = \int_c^x f(t)\,dt$ then:

$$A'(x) = \frac{d}{dx}\int_c^x f(t)\,dt$$

$$A'(x) = \frac{d}{dx}(F(x) - F(c))$$

$$A'(x) = \frac{d}{dx}(F(x)) - \frac{d}{dx}(F(c))$$



How can we further simplify $\frac{d}{dx}(F(x)) - \frac{d}{dx}(F(c))$?

constant = 0

$\boxed{f(x) + 0}$

Use the Fundamental Theorem of Calculus to evaluate each expression and compare the results.

a. $\frac{d}{dx}\int_3^x (3x-5)dx$ $\boxed{3x-5}$

b. $\int_3^x \frac{d}{dx}(3x-5)dx = \int_3^x 3 = 3x\Big|_3^x = \boxed{3x-9}$

$f(x) - f(3)$
$3x - 3(3)$

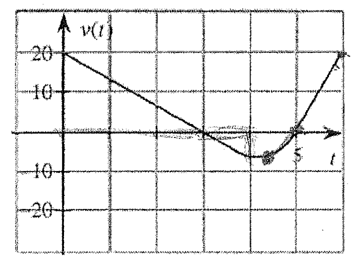
c. $\frac{d}{dx}\int_3^5 (3x-5)dx$

$\boxed{0}$

d. $\int \frac{d}{dx}(3x-5)dx$

$\boxed{3x+C}$

The graph at right shows the velocity (in miles per hour) of a car during a road trip. At time $t = 0$, the car was on the Golden Gate Bridge heading north.



$(5,0)$ $(6,20)$

$m = \frac{20}{1} = m = 20$

$y = mx + b$

$0 = 20(5) + b$

$-100 = b$

$y = 20x - 100$

a. Find a function for $v(t)$.

$$v(t) = \begin{cases} \frac{-20}{3}t + 20 & 0 \le t \le 4.5 \\ 20t - 100 & 4.5 < t \le 5 \end{cases}$$

b. How far north has the car traveled at 3 hours?
At 4 hours?

$\frac{1}{2}(3)(20) = \boxed{30 \text{ miles}}$

$\frac{1}{2} \cdot b \cdot h$

$30 + \frac{1}{2}(1)\left|\frac{20}{3}(4) + 20\right| = \boxed{\frac{100}{3}}$

c. Explain what happened to the car between $3 \le t \le 5$ hours.

Changed direction

d. Set up an integral to represent the displacement from $0 \le x \le 5$.

$\int_0^5 v(t)\, dt$

e. Set up an integral to represent the total distance from $0 \le x \le 5$.

Total dist $= \int_0^5 |v(t)|\, dt$

need only (+) 1M