AP Calculus AB Chapter 5 Test 2 - Review

*Review definition of continuity

*Review how to use the trapezoid rule using a test of values.

1. If $\int_0^a \cos(\cancel{x+5}\ x+\pi)\,dx = c$, then $\int_{-a}^a \cos(\cancel{x+5}\ x+\pi)\,dx$ = ? $\boxed{2c}$

Evaluate each:

2. $\int (x^5 - 6x^2 + x - 1)dx$ $= \boxed{\frac{1}{6}x^6 - 2x^3 + \frac{1}{2}x^2 - x + C}$

3. $\int (\sqrt{x} + \frac{1}{x^5})dx$ $\boxed{\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{4x^4} + C}$

4. $\int \left(1 - \frac{1}{\sqrt[3]{x^4}}\right)dx$ $= x - \frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} + C = \boxed{x + \frac{3}{\sqrt[3]{x}} + C}$

5. $\int (x^3 - 4\sin x)\,dx$ $= \boxed{\frac{1}{4}x^4 + 4\cos x + C}$

6. $\int_1^4 \frac{x^3 - 8}{\sqrt{x}}dx$ See work on back $= \boxed{\frac{142}{7}}$

7. $\int_0^2 (4x^3 + x - 1)dx$ $x^4 + \frac{1}{2}x^2 - x \Big|_0^2 = \boxed{16}$

8. $\int_{-\pi}^{\pi} \sin x\,dx$ $= \boxed{0}$

Use u-substitution to solve.

9. $\int x(x+1)^{10}dx$ let $u = x+1$, $du = dx$ and $x = u-1$ $\int (u-1)u^{10}du = \int (u^{11} - u^{10})du = \boxed{\frac{(x+1)^{12}}{12} - \frac{(x+1)^{11}}{11} + C}$

10. $\int x\sqrt{x-2}\,dx$ $\boxed{\frac{2(x-2)^{\frac{5}{2}}}{5} + \frac{4(x-2)^{\frac{3}{2}}}{3} + C}$ work on back

11. $\int (2x-5)^{\frac{2}{3}}dx$ $\boxed{\frac{3(2x-5)^{\frac{5}{3}}}{10} + C}$

12. $\int \sin 4x\,dx$ $= \boxed{-\frac{1}{4}\cos(4x) + C}$

13. If $\int_{-2}^{k}(4x+1)dx = 30$, find k.

$2x^2 + x \Big|_{-2}^{k} = 30 \rightarrow (2k^2 + k) - (8-2) = 30 \rightarrow 2k^2 + k - 6 = 30$ $\boxed{k = 4}$

Find $\frac{dy}{dx}$

14. y = $\int_1^{2x} \frac{1}{t^8}dt$ $dy = 2\,dx$

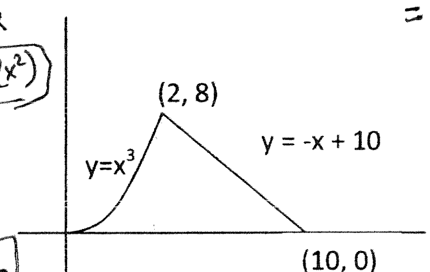
Plug in for ANS, mult by deriv to apply chain rule $\boxed{\frac{1}{(2x)^8} \cdot 2}$

$\frac{1}{(2x)^8} \cdot 2 = \frac{2}{256x^8} = \boxed{\frac{1}{128x^8}}$

15. y = $\int_{x^2}^1 \sin t\,dt = -\int_1^{x^2}$ $dy = 2x\,dy$

$-\sin x^2 \cdot 2x = \boxed{-2x\sin(x^2)}$

16. $\int_1^{x^3} \sqrt{t^2+1}\,dt$ $\rightarrow dy = 3x^2dx$ $= \sqrt{(x^3)^2 + 1} \cdot 3x^2 = \boxed{3x^2\sqrt{x^6+1}}$



17. Find the area under the 2-curves shown

$\int_0^2 x^3dx + \int_2^{10}(-x+10)dx = 4 + 32 = \boxed{36}$

18. y = 6 - 2xy

WORK ON BACK a) Find $\frac{dy}{dx}$ $= \frac{-2y}{2x+1}$ b) Find $\frac{d^2y}{dx^2}$ $= \frac{8y}{(2x+1)^2}$

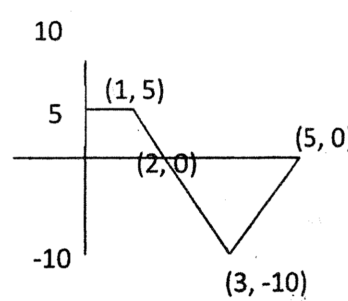
19. The graph of the velocity of a particle is given by: Speed = $|v(t)|$

a) When will it have the maximum speed? at $t=3$ $v(t) = -10$

b) When is the acceleration positive? when slope = (+) so $\boxed{(3,5)}$

c) What is the total distance traveled?

d) Where is the particle when t = 5?



ⓒ Total Area under graph

$5 + 5 + 15$; $5 + \frac{5}{2} + 15 = \boxed{\frac{45}{2}}$

ⓓ displacement @ t=5

$5 + \frac{5}{2} - 15 = \boxed{\frac{-15}{2}}$

(6) $\frac{x^3-8}{\sqrt{x}} = \frac{x^3}{\sqrt{x}} - \frac{8}{\sqrt{x}} = \frac{x^3}{x^{\frac{1}{2}}} - \frac{8}{x^{\frac{1}{2}}}$

$= \int_1^4 x^{\frac{5}{2}} - 8x^{-\frac{1}{2}} = \frac{2}{7}x^{\frac{7}{2}} - 16x^{\frac{1}{2}} \Big|_1^4$

$= \frac{32}{7} - \frac{-110}{7} = \boxed{\frac{142}{7}}$

(10) let $u = x-2$
$du = dx$ so $\int (u+2)\sqrt{u}\, du$
and $x = u+2$ $= \int \left(u^{3/2} + 2u^{\frac{1}{2}}\right) du$
$= \frac{2}{5}u^{\frac{5}{2}} + \frac{4}{3}u^{\frac{3}{2}} + C$
$= \boxed{\frac{2}{5}(x-2)^{\frac{5}{2}} + \frac{4}{3}(x-2)^{3/2} + C}$

(11) let $u = 2x-5$
$du = 2dx$
$\int u^{2/3} \cdot \frac{1}{2} du$
$= \frac{1}{2} \cdot \frac{3}{5} u^{\frac{5}{3}} + C$
$\boxed{\frac{3}{10}(2x-5)^{5/3} + C}$

(18) $y = 6 - 2xy$

~~$y + 2xy = 6$~~

~~$y(1+2x) = 6$~~

$dy = 0 - (2x\,dy + y(2dx))$

$dy = -2x\,dy - 2y\,dx$

$2x\,dy + dy = -2y\,dx$

$dy(2x+1) = -2y\,dx$

$\boxed{\frac{dy}{dx} = \frac{-2y}{2x+1}}$

OR

$y' = 0 - 2(xy' + y)$

$y' = -2xy' + 2y$

$2xy' + y' = -2y$

$y'(2x+1) = -2y$

$y' = \frac{-2y}{2x+1}$

(b) $\frac{d^2y}{dx^2} = \frac{(2x+1)\left(-2\frac{dy}{dx}\right) - (-2y \cdot 2)}{(2x+1)^2}$

$= \frac{(2x+1)\left(-2\left(\frac{-2y}{2x+1}\right)\right) - (-4y)}{(2x+1)^2}$

$= \frac{4y+4y}{(2x+1)^2} = \boxed{\frac{8y}{(2x+1)^2}}$

OR

$y'' = \frac{(2x+1)(-2y') - (-2y)(2)}{(2x+1)^2}$

$y'' = \frac{(2x+1)\left(-2\left(\frac{-2y}{2x+1}\right)\right) - (-4y)}{(2x+1)^2}$

$y'' = \frac{4y+4y}{(2x+1)^2} = \boxed{\frac{8y}{(2x+1)^2}}$