KEY

Chapter 6 Test Review

1. Rojito is about to haul up a 15 m length of rope hanging from the top of the Joe Raya gym. (He used this rope to haul up the physics project of his friend, Friend, to put on top of the gym for all the world to see… well just EDHS) How much work will it take to haul up the rope if it weighs 1.2 N/m? Assume uniform weight.

2. Suppose $f(x) = xe^x$ and $g(x) = \cos x$
   a) Find the area of the region in the first quadrant bounded by the graphs of f(x), g(x) and x = 0.

   b) The volume of the region in part (a) about the x-axis.

   c) Suppose the region in part (a) is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares with bases extending from y = f(x) to y = g(x). Find the volume of this solid.

3. $f(x) = 5x^2 + 12$. Determine a c such that f(c) is the average value on (-4, 8).

4. Find the area between the x-axis and $y = e^{2x}$ on the interval [0, 1].

5. Find the area between $y = 2\sin x$ and $y = \sin 2x$ on $[0, \pi]$

6. The volume V in liters of air in the lungs during a five-second respiratory cycle is approximated by the model $V = 0.1729t + 0.1522t^2 - 0.0374t^3$ where t is the time in seconds. Approximate the average volume of air in the lungs during one cyle.

KEY

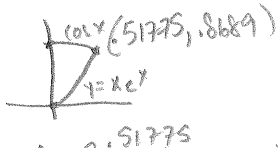
① 

$(15)(1.2) = 18$ N Total weight

$18 - 1.2x$ ← weight loss per m

$W = \int_0^{15} (18 - 1.2x)\,dx$ F·d

$W = 135$ Nm

② 

$A = \int_0^{.51775} (\cos x - xe^x)\,dx$

$A = .3043\ u^2$

(b) $V = \pi \int_0^{.51775} [(\cos x)^2 - (xe^x)^2]\,dx$

$V = .3713\pi$

(c) $V = A(x)dx = \int_0^{.51775} (\cos x - xe^x)^2\,dx$

where $x = g(x) - f(x)$
$x = \cos x - xe^x$

$A = .2219\ u^2$

③ $\int_a^b f(x)dx = f(c)(b-a)$

$\int_{-4}^{8} (5x^2 + 12)\,dx$

$\frac{5}{3}x^3 + 12x \Big|_{-4}^{8} = \left[\frac{2848}{3} - \frac{-464}{3}\right]$

$= 1104$

thus $1104 = f(c)(8 - -4)$

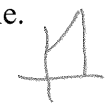
$92 = f(c)$

Now $5c^2 + 12 = 92$

$c^2 = 16$

$c = \pm 4$

ONLY 4 since on the interval [-4, 8]

④ 

$A = \int_0^1 e^{2x}\,dx$

$= \frac{1}{2}(e^2 - 1)$

OR 3.195

⑤ 

$A = \int_0^{\pi} (2\sin x - \sin 2x)\,dx$

$A = -2\cos x + \frac{1}{2}\cos 2x$

$A = \left(-2(-1) + \frac{1}{2}\right) - \left(-2 + \frac{1}{2}\right)$

$\left(2\frac{1}{2}\right) - \left(-1\frac{1}{2}\right) = [4$

⑥ $V = \frac{1}{5-0}\int_0^5 .1729t + .1522t^2 - .0374t^3\,dt$

$V = .5318\ l^3$