Volume Disk/Washer/Cross Sectional Additional Examples Video

1. Find the volume of the solid that results when the region y = $2\sqrt{x+3}$, y = 0, x = 0 and x = 2 is revolved about the x-axis.

2. Find the volume of the solid that results when the region y = $\sqrt{4-x}$, x = 0, y = 0 and y = 2 is revolved about the y-axis.

3. Find the volume of the solid that results when the region bounded by y = 7 – x^2, x = -2 and x = 2 is revolved about the x-axis

4. Find the volume of the solid that results when the region bounded by y = $6e^{-2x}$, y = $6+4x-2x^2$, x = 0 and x = 1 is revolved about the line y = -2

5. Find the volume of the solid of revolution generated by revolving the region bounded by $y = 1 - x$, $y = 0$, and $x = 0$ about:

a) the x–axis

b) the y–axis

c) the line $y = -1$

6. Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$, $y = x^2 - 1$, with cross sections perpendicular to the axis and are:

a) Squares

b) rectangles that each have a height of 1

c) equilateral triangles

d) semi-circles

Answers:

1. 32π 2. $256\pi/15$ 3. $2012\pi/15$ 4. 64.2556π 5. a) 1.047 b) 1.047 c) 4.189

6. a) 8.1 b) 4.5 c) $\frac{81\sqrt{3}}{40}$

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the x-axis.

1) $y = 2\sqrt{x+3},\ y = 0,\ x = 0,\ x = 2$

2) $y = x^2,\ y = 0,\ x = 2$

3) $y = \sqrt{\sin x},\ y = 0,\ x = 0,\ x = \pi$

4) $x = \sqrt{1-y},\ x = -\sqrt{1-y},\ y = 0$

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the y-axis.

5) $x = -y^2 + 4,\ x = 0,\ y = 0,\ y = 2$

6) $x = \dfrac{2}{y},\ x = 0,\ y = \dfrac{2}{5},\ y = 6$

Answers to Finding Volumes of Solids of Revolution

1) $\pi \displaystyle\int_0^2 \left(2\sqrt{x+3}\right)^2 dx = 32\pi \approx 100.531$

2) $\pi \displaystyle\int_0^2 \left(x^2\right)^2 dx = \frac{32}{5}\pi \approx 20.106$

3) $\pi \displaystyle\int_0^{\pi} \left(\sqrt{\sin x}\right)^2 dx = 2\pi \approx 6.283$

4) $\pi \displaystyle\int_{-1}^{1} \left(-x^2 + 1\right)^2 dx = \frac{16}{15}\pi \approx 3.351$

5) $\pi \displaystyle\int_0^2 \left(-y^2 + 4\right)^2 dy = \frac{256}{15}\pi \approx 53.617$

6) $\pi \displaystyle\int_{\frac{2}{5}}^{6} \left(\frac{2}{y}\right)^2 dy = \frac{28}{3}\pi \approx 29.322$

7) $\pi \displaystyle\int_{-\frac{\pi}{6}}^{0} \left(2\sec y\right)^2 dy = \frac{4\sqrt{3}}{3}\pi \approx 7.255$

8) $\pi \displaystyle\int_{-2}^{2} \left(-y^2 + 4\right)^2 dy = \frac{512}{15}\pi \approx 107.233$

5. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $y = 6\mathbf{e}^{-2x}$ and $y = 6 + 4x - 2x^2$ between $x = 0$ and $x = 1$ about the line $y = -2$.

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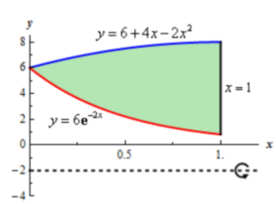
Hide Solution

We need to start the problem somewhere so let's start "simple".

Knowing what the bounded region looks like will definitely help for most of these types of problems since we need to know how all the curves relate to each other when we go to set u the area formula and we'll need limits for the integral which the graph will often help with.

Here is a sketch of the bounded region with the axis of rotation shown.



Find the volume of the solid that results when the region bounded by $y = 6e^{-2x}$, $y = 6 + 4x - 2x^2$, x = 0 and x = 1 is revolved about the line y = -2

$$\text{Inner Radius} = 2 + 6\mathbf{e}^{-2x} \qquad \text{Outer Radius} = 2 + 6 + 4x - 2x^2 = 8 + 4x - 2x^2$$

$$\begin{aligned} A(x) &= \pi\left[(\text{Outer Radius})^2 - (\text{Inner Radius})^2\right] \\ &= \pi\left[\left(8 + 4x - 2x^2\right)^2 - \left(2 + 6\mathbf{e}^{-2x}\right)^2\right] \\ &= \pi\left(60 + 64x - 16x^2 - 16x^3 + 4x^4 - 24\mathbf{e}^{-2x} - 36\mathbf{e}^{-4x}\right) \end{aligned}$$

integral for the volume and evaluate it.

n see that the “first” ring in the solid would occur at $x = 0$ and the “last” ring would occur at $x = 1$. Our limits are then : $0 \le x \le 1$.

$$\begin{aligned} V &= \int_0^1 \pi\left(60 + 64x - 16x^2 - 16x^3 + 4x^4 - 24\mathbf{e}^{-2x} - 36\mathbf{e}^{-4x}\right)\,dx \\ &= \pi\left(60x + 32x^2 - \frac{16}{3}x^3 - 4x^4 + \frac{4}{5}x^5 + 12\mathbf{e}^{-2x} + 9\mathbf{e}^{-4x}\right)\Bigg|_0^1 = \boxed{\left(\frac{937}{15} + 12\mathbf{e}^{-2} + 9\mathbf{e}^{-4}\right)\pi} \end{aligned}$$

8. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $y = 2x + 1$, $x = 4$ and $y = 3$ about the line $x = -4$.

$$V = \int_3^9 \pi\left(\frac{207}{4} - \frac{7}{2}y - \frac{1}{4}y^2\right)\,dy = \pi\left(\frac{207}{4}y - \frac{7}{4}y^2 - \frac{1}{12}y^3\right)\Bigg|_3^9 = \boxed{126\pi}$$

2. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $y = 7 - x^2$, $x = -2$, $x = 2$ and the x-axis about the x-axis.

$$V = \int_{-2}^2 \pi\left(49 - 14x^2 + x^4\right)\,dx = \pi\left(49x - \frac{14}{3}x^3 + \frac{1}{5}x^5\right)\Bigg|_{-2}^2 = \boxed{\frac{2012}{15}\pi}$$

4. Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $y = 2x^2$ and $y = x^3$ about the x-axis.

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$$V = \int_0^2 \pi \left(4x^4 - x^6\right)\, dx = \pi \left(\frac{4}{5}x^5 - \frac{1}{7}x^7 \right) \Bigg|_0^2 = \boxed{\frac{256}{35}\pi}$$