**AP Calc Chapter 7 Review – All**

Find the limit of each. Use L'Hopital if needed.

1. $\lim_{x\to 0}\frac{\sin 3x}{\sin 4x}$ 2. $\lim_{x\to 0}\frac{x-\sin x}{x^3}$ 3. $\lim_{x\to\infty}\frac{5x^3-4x^2+1}{7x^3+2x-6}$ 4. $\lim_{x\to\infty} xe^{-2x}$

Find the derivative of each.

5. $y = \ln^3 x$ 6. $f(x) = \frac{x^2}{\ln x}$ 7. $y = 3^{-2x}$ 8. $f(x) = \sin^{-1}(2x)$

9. $f(x) = 3\arccos(5x)$ 10. $f(x) = 4^{x-5}$

Integrate each.

11. $\int\frac{4}{\sqrt{25-x^2}}\,dx$ 12. $\int\frac{1}{36+4x^2}\,dx$ 13. $\int xe^{3x^2+1}\,dx$ 14. $\int 6^x dx$ 15. $\int\frac{e^x}{1+e^x}\,dx$

16. Find $(f^{-1})'(a)$ for $f(x) = \frac{2}{x} - e^x$ at $a = 1$.

17. The waitress pours coffee into your cup at 10:00 am. The coffee is $180^0$ when freshly poured and after 2 minutes in a room at $70^0$F, the coffee has cooled to $165^0$F. Find the temperature at any time t and find the time at which the coffee is $120^0$F.

Answers

① $\frac{0}{0}$ Ind
L'Hop $\frac{\cos 3x\cdot 3}{\cos 4x\cdot 4} = \boxed{\frac{3}{4}}$

② $\frac{0-0}{0} = \frac{0}{0}$
L'Hop $\frac{1-\cos x}{3x^2} = \frac{1-1}{0} = \frac{0}{0}$
L'Hop $\frac{\sin x}{6x} = \frac{0}{0}$ L'Hop $\frac{\cos x}{6} = \boxed{\frac{1}{6}}$

③ $\boxed{\frac{5}{7}}$

④ $\frac{\infty}{\infty}$ L'Hop
$\lim_{x\to\infty}\frac{x}{e^{2x}} \to \frac{1}{e^{2x}\cdot 2}$
$= \boxed{0}$

⑤ $y' = 3(\ln x)^2\cdot\frac{1}{x}$
$\boxed{y' = \frac{3(\ln x)^2}{x}}$

⑥ $f'(x) = \frac{\ln x\,(2x) - x^2\cdot\frac{1}{x}}{(\ln x)^2}$
$\boxed{f'(x) = \frac{2x\ln x - x}{(\ln x)^2}}$

⑦ $y' = 3^{-2x}\cdot\ln 3\cdot(-2)$
$\boxed{y' = -2\cdot 3^{-2x}\ln 3}$

⑧ $\boxed{f'(x) = \frac{1}{\sqrt{1-4x^2}}\cdot 2}$

⑨ $f'(x) = \frac{-3}{\sqrt{1-25x^2}}\cdot 5$
$\boxed{f'(x) = \frac{-15}{\sqrt{1-25x^2}}}$

⑩ $f'(x) = 4^{x-5}\ln 4\cdot 1$
$\boxed{f'(x) = (\ln 4)\,4^{x-5}}$

⑪ $4\int\frac{1}{\sqrt{25-x^2}}\,dx$
$= \boxed{4\sin^{-1}\left(\frac{x}{5}\right)+C}$

⑫ $a=6,\ 2x$
$s.$
$\frac{1}{6}\tan^{-1}\left(\frac{2x}{6}\right)\cdot\frac{1}{2}+C$
$\boxed{\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)+C}$

⑬ let $u = 3x^2+1$
$du = 6x\,dx$
$\frac{du}{6x} = dx$
$\int xe^u\cdot\frac{du}{6x} = \frac{1}{6}\int e^u du$
$\frac{1}{6}e^u + C = \boxed{\frac{1}{6}e^{3x^2+1}+}$

(14) $\int 6^x\,dx = \boxed{\frac{6^x}{\ln 6} + C}$

(15) $\int \frac{e^x}{1+e^x}\,dx$

$= \boxed{\ln|1+e^x| + C}$

(16) $f'(x) = -2x^{-2} - e^x$

$f'(x) = \frac{-2}{x^2} - e^x$

$x = \frac{2}{y} - e^y$ @ $x = 1$

$\left[1 = \frac{2}{y} - e^y\right]$ y

$y = 2 - ye^y$ ← solve by graphing

$y = .67483$

$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$

$f'(.67483) =$

$-6.303$

Thus $\frac{1}{-6.303} = \boxed{-.158}$

(17) $T - T_s = (T_0 - T_s)e^{kt}$

$165 - 70 = (180 - 70)e^{k(2)}$

$95 = (110)e^{2k}$

$.8636 = e^{2k}$

$\frac{\ln .8636}{2} = k$

$\boxed{k = -.0733}$

$\boxed{T = 110e^{-.0733t} + 70}$

$120 = 110e^{-.0733t} + 70$

↓

$t = 10.76$ min, so $\boxed{10:10 \text{ \& } 45.6 \text{ sec}}$