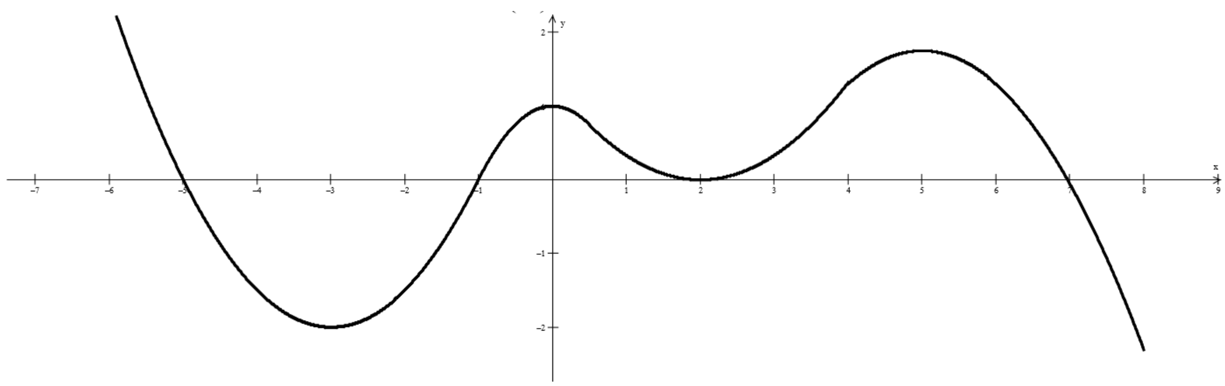
AP Calculus Test Review – Max, Min, Concavity and Asbolutes

The figure shows the graph of *f′*, the derivative of the function *f,* on the closed interval -6 ≤ x ≤ 8. The graph of *f′* has horizontal tangent lines at x = -3, x = 0, x = 2 and x = 5. The function *f* is twice differentiable with *f(2) = -4.*



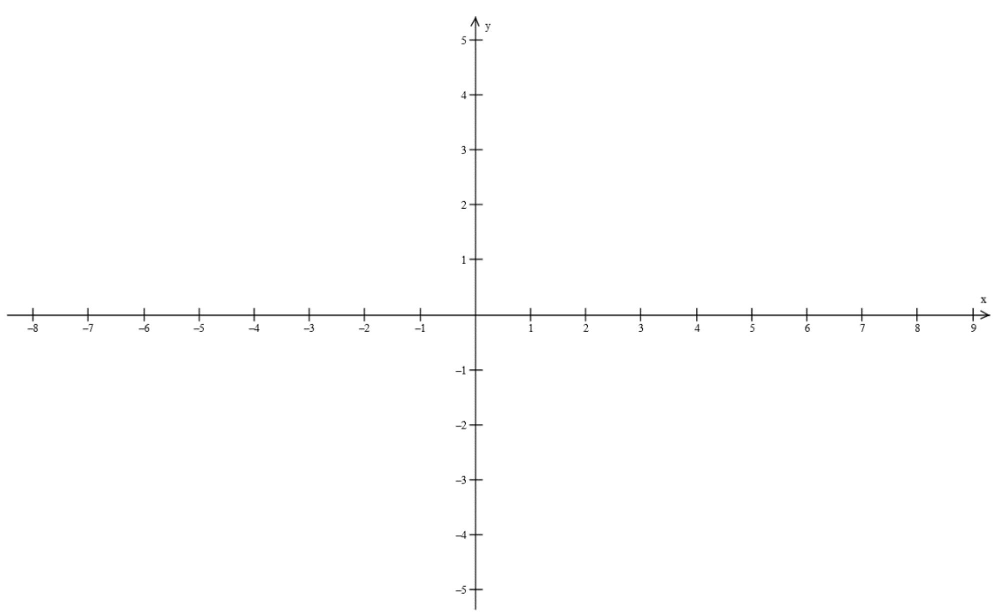
(a) Find the x-coordinate of each of the points of inflection of the graph of f. Give a reason for your answer.

(b) At what value of x does *f* attain its absolute minimum AND maximum value on the closed interval -6 ≤ x ≤ 8? Show the work that leads to your answers.

(c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

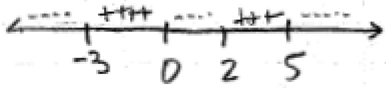
(d) Find all intervals on which the graph of *f* is concave up and has a positive slope. Explain your reasoning.

(e) For -6 ≤ x ≤ 8, sketch a possible graph of *f*.



⑦ (a) POI when $f''(x)=0$ so need Horiz Tan of f'

Possible @ $x=-3, 0, 2, 5$ ← Test slopes



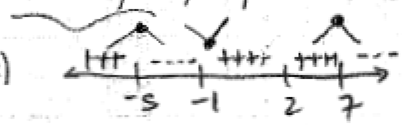
f''

Thus slopes of f' change @ $\boxed{x=-3,\ 0,\ 2 \ \&\ 5 \text{ All are points of inflect}}$

(b) Need $f'(x)=0$ for CRIT PTS

CRIT PTS @ $x=-5, -1, 2, 7$ and endpts $-6, 8$

Test $f'(x)$



f' changes from increasing/decrea @ $-5, -1, 7$

Rel Max @ $x=-5$, $x=7$ Rel min @ $x=-1$

ABS MAX @ $x=7$ ← greater (+) AREA from $\int_0^7$ than $\int_{-5}^0$ and $\int_0^8$

ABS MIN @ $x=-1$ ← least Area

(c) need slope @ $x=2$ → $g'(x) = x f'(x) + f(x)$

$g'(2) = 2f'(2) + f(2) = 2(0) + -4 = -4 = m$

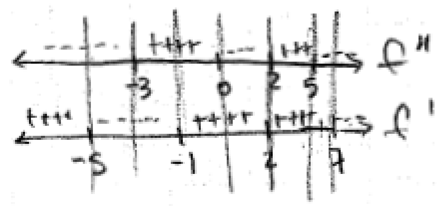
$g(2) = 2(f(2)) = 2(-4) = 8$

$\boxed{y-2=-4(x+8)}$

(d) Concave up when $f''>0$

positive slope when $f'>0$ (Increasing)

$\boxed{\text{Thus Both (+) } (-1,0)\cup(2,5)}$



(e)