

Related Rates

1. A camera on the ground 300 meters away from the Launchpad records a hot air balloon rising at a rate of 10 meters/sec. How fast is the camera's angle of elevation changing when the hot air balloon is 400 meters high?
2. A light on the ground 120 feet from a building is shining at a 6-ft man walking away from the streetlight and toward the building at a rate of 8 ft/sec. How fast is his shadow on the building becoming shorter when he is 40 ft. from the building?
3. A ladder 15 ft long leans against a house. If the foot of the ladder is moving away from the house at the rate of 2 ft/sec, find the rate of change of the top of the ladder on the side of the house when the foot of the ladder is 12 ft from the house.

Real World Rate of Change

1. The rate at which people enter the OC Fair on a given day is modeled by the function E defined by

$$E(t) = \frac{16000}{t^2 - 24t + 160}$$

The rate at which people leave the OC Fair on the same day is modeled by the function L defined by

$$L(t) = \frac{10000}{t^2 - 38t + 370}$$

Both E(t) and L(t) are measured in people per hour and time t is measured in **HOURS AFTER MIDNIGHT**.

These functions are valid between 9 a.m. and 11 p.m., the hours when the OC Fair is open. At 9 a.m., there are already 200 employees in the Fairgrounds.

- a) How many people have entered the OC Fair from 9 a.m. to 6 p.m.?
- b) How many people are in the Fairgrounds from 9 a.m. to 6 p.m.?
- c) if the OC Fair decides to charge \$15 for admission until 6 p.m. and \$10 for admission after 6 p.m., how much revenue will they generate this day?
- d) Is the number of people at the fair increasing or decreasing at 7 p.m.? Explain your answer.
- e) What is the average rate of people entering the Fairgrounds from 4 p.m. until 7 p.m.?

f) What is the average rate of change of people in the OC Fairgrounds from 4 p.m. until 7 p.m.? Explain the meaning of your answer in the context of this problem.

g) At what time is the number of people in the OC Fairgrounds a maximum? Justify your answer.

h) If $H(t) = \int_9^t (E(x) - L(x)) dx$, explain the meaning of $H(13)$ and $H'(13)$.

2. A water tank at Camp Calculus contains 1200 gallons of water at time $t = 0$. During this time interval $0 \leq t \leq 18$ hours, water is pumped into the tank at a rate of $W(t) = 94\sqrt{t}\sin^2\left(\frac{t}{5}\right)$ gallons per hour.

During the same time interval, water is removed from the tank at a rate of $R(t) = 270\sin^2\left(\frac{t}{4}\right)$ gallons per hour.

a) How many gallons of water are in the tank at time $t = 18$?

b) Is the amount of water in the tank increasing or decreasing at time $t = 14$? Explain.

c) At what time t , for $0 \leq t \leq 18$ is the amount of water in the tank a minimum? Show the work that leads to your answer.

d) At what time t , for $0 \leq t \leq 18$ is the amount of water in the tank a maximum? Show the work that leads to your answer.

e) For the interval $[3, 14]$, what is the average rate at which the amount of water in the tank is changing?

f) For $t > 18$, water is no longer pumped into the tank, but it continues to be removed from the tank according to $R(t)$. If k is the time at which the tank becomes empty, write but do not solve an expression involving an integral that can be used to find the value of k .