

1. A company has started selling a new type of smartphone at the price of $\$110 - 0.05x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost $\$50$ and the labor and overhead for the plant cost $\$6000$ per day. How many smartphones should the company manufacture and sell per day to maximize profit? (Profit = Revenue – Cost)
2. A rancher wants to construct two identical corrals using 800 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?
3. Let $f(x) = \begin{cases} 3x^2 + 1 & x < 0 \\ 1 - \sin x & x \geq 0 \end{cases}$. Which of the following statements is true about f ?
- I. $\lim_{x \rightarrow 0} f(x)$ exists.
 - II. f is continuous at $x = 0$.
 - III. f is differentiable at $x = 0$.
- (A) None
(B) I only
(C) II only
(D) I and II only
(E) I, II, and III
4. The radius of a sphere is decreasing at a constant rate of 3 centimeters per second. In terms of the surface area of the sphere S , what is the rate of change of the volume of the sphere, in cubic centimeters per second
- (A) S
(B) $3S$
(C) $-3S$
(D) $3\pi S$
(E) $-3\pi S$

5. Let f be the function defined below. Which of the following statements about f are true?

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 1 & \text{if } x = 3 \end{cases}$$

- I. f has a limit at $x = 3$.
- II. f is continuous at $x = 3$.
- III. f is differentiable at $x = 3$.

- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I, II, and III only

6. $\lim_{h \rightarrow 0} \frac{\ln(2+h) - \ln 2}{h} =$ (A) 0 (B) $\ln 2$ (C) $\frac{1}{2}$ (D) $\frac{1}{\ln 2}$ (E) ∞

7. $\lim_{h \rightarrow 0} \frac{\sin(\frac{\pi}{2} + h) - 1}{h} =$ (A) -1 (B) 0 (C) 1 (D) ∞ (E) DNE

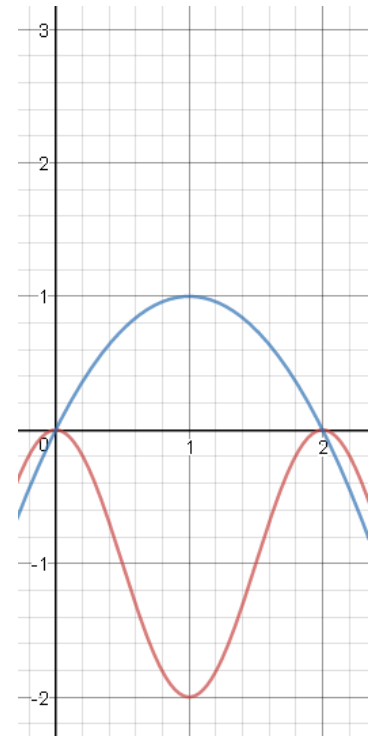
8. The region R is bounded by the curves $f(x) = \cos(\pi x) - 1$ and $g(x) = x(2 - x)$.

a) Find the area of region R.

b) A solid has base R, and each cross section perpendicular to the x-axis is an isosceles right triangle whose hypotenuse lies in R. Set up but do not evaluate, an integral for the volume of this solid.

c) A solid has base R, and each cross section perpendicular to the x-axis is a semi-circle whose diameter lies in R. Set up but do not evaluate, an integral for the volume of this solid.

d) Set up, but do not evaluate an integral for the volume of the solid formed when R is rotated around the line $y = 3$.



9. The temperature of the water in a swimming pool at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Celsius and t is measured in minutes. At time $t = 0$, the water in the pool is 20°C . The water is heated for 45 minutes, beginning at time $t = 0$. Select values of $W(t)$ for the first 30 minutes are shown in the table.

t (minutes)	0	7	12	20	30
$W(t)$ $^\circ\text{C}$	20.0	20.8	22.7	24.6	25.8

a) Using the values in the table, estimate $W'(10)$. Show the computation that lead to your answer. Using the correct units, interpret your answer in the context of this problem.

b) Use the data in the table to evaluate $\int_0^{30} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{30} W'(t) dt$ in the context of this problem.

c) For $0 \leq t \leq 30$, the average temperature of the water in the pool is $\frac{1}{30} \int_0^{30} W(t) dt$.

- Use right Riemann Sum with four subdivisions indicated by the data in the table to approximate $\frac{1}{30} \int_0^{30} W(t) dt$. Does this approximation underestimate or overestimate the average temperature of the water of these 30 minutes?

- Use left Riemann Sum with four subdivisions indicated by the data in the table to approximate $\frac{1}{30} \int_0^{30} W(t) dt$. Does this approximation underestimate or overestimate the average temperature of the water of these 30 minutes?

- Use midpoint Riemann Sum with four subdivisions indicated by the data in the table to approximate $\frac{1}{30} \int_0^{30} W(t) dt$. Does this approximation underestimate or overestimate the average temperature of the water of these 30 minutes?

- Use trapezoid Riemann Sum with four subdivisions indicated by the data in the table to approximate $\frac{1}{30} \int_0^{30} W(t) dt$. Does this approximation underestimate or overestimate the average temperature of the water of these 30 minutes?

d) For $30 \leq t \leq 45$, the function W that models the water temperature has first derivative given by $W'(t) = 0.132\sqrt{t} \cos(0.05t)$. Based upon this model, what is the temperature of the water at time $t = 45$?