AP Calculus - Chapter 2 Test 1 - Review Problems

1.

x	3.9	3.99	3.999	3.9999	4.0001	4.001	4.01	4.1
$f(x)$	5	-25	125	-625	5.9999	5.999	5.99	5.9

The table above gives values of a function f at selected values of x. Which of the following conclusions is supported by the data in the table?

(A) $\lim_{x\to4} f(x) = 6$

(B) $\lim_{x\to4^-} f(x) = 6$

(C) $\lim_{x\to4^+} f(x) = 6$

(D) $\lim_{x\to6^+} f(x) = 4$

2.

If f is the function defined by $f(x) = \frac{x-9}{\sqrt{x}-3}$, then $\lim_{x\to9} f(x)$ is equivalent to which of the following?

(A) $\lim_{x\to9} (\sqrt{x}-3)$

(B) $\lim_{x\to9} (\sqrt{x}+3)$

(C) $\lim_{x\to9} \left(\frac{x^2-81}{x-9}\right)$

(D) $\frac{\lim_{x\to9}(x-9)}{\lim_{x\to9}(\sqrt{x}-3)}$

$\frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \frac{\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{(x-9)}}$

$\lim_{x\to9} \sqrt{x}+3$

3.Find each limit:

$\lim_{x\to\infty} \frac{20}{10+e^{-x}}=$ $\frac{20}{10+\frac{1}{e^x}\to0} = \boxed{2}$

$\lim_{x\to0} \frac{cosx}{sinx-e^x}=$ $\frac{1}{0-1} = \boxed{-1}$ $e^0=1$

$\lim_{x\to0} \frac{7x^5+5x^2+12x}{3x^5+4x}$ is $\frac{0}{0}$ $\frac{\cancel{x}(7x^4+5x+12)}{\cancel{x}(3x^4+4)}$ $\frac{12}{4} = \boxed{3}$

4.

If $f(x) = \frac{\sin x-1}{\cos^2 x}$, then $\lim_{x\to\frac{\pi}{2}} f(x)$ is equivalent to which of the following?

(A) $\lim_{x\to\frac{\pi}{2}} \frac{-1}{1+\sin x}$

(B) $\lim_{x\to\frac{\pi}{2}} \frac{\sin x-1}{1+\sin^2 x}$

(C) $\lim_{x\to\frac{\pi}{2}} \sec x$

(D) $\lim_{x\to\frac{\pi}{2}} (\tan x - \sec x)$

$\frac{\sin x-1}{1-\sin^2 x} = \frac{\sin x-1}{(1-\sin x)(1+\sin x)} = \frac{-\cancel{(1-\sin x)}}{\cancel{(1-\sin x)}(1+\sin x)}$

$= \frac{-1}{1+\sin x}$

5.

If f is the function defined by $f(x) = \frac{\frac{1}{x}-1}{x-1}$, then $\lim\limits_{x\to1} f(x)$ is equivalent to which of the following?

$\frac{\frac{1}{x}-1}{x-1} \cdot \frac{x}{x} = \frac{1-x}{(x-1)x} = \frac{-\cancel{(x-1)}}{\cancel{(x-1)}x}$

$= \frac{-1}{x}$

(A) $\lim\limits_{x\to1}\left(-\frac{1}{x}\right)$

(B) $\lim\limits_{x\to1}\left(\frac{1}{x^2}-1\right)$

(C) $\lim\limits_{x\to1}\left(\frac{x-1}{x-1}\right)$

(D) $\frac{\lim\limits_{x\to1}\left(\frac{1}{x}-1\right)}{\lim\limits_{x\to1}(x-1)}$

6.

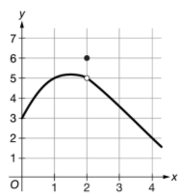
The function g is given by $g(x) = \frac{7x-26}{x-5}$. The function h is given by $h(x) = \frac{3x+14}{2x+1}$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for $0 < x < 5$, what is $\lim\limits_{x\to2} f(x)$? $= \boxed{4}$ Squeeze thm

$\frac{7(2)-26}{2-5} = \frac{-12}{-3} = 4$

$\frac{3(2)+14}{2(2)+1} = \frac{20}{5} = 4$

7.

$$f(x) = \begin{cases} -x^2+3x+3 & \text{for} \quad x<2 \\ 6 & \text{for} \quad x=2 \\ 8-\frac{3}{2}x & \text{for} \quad x>2 \end{cases}$$



Let f be the piecewise function defined above. Also shown is a portion of the graph of f. What is the value of $\lim\limits_{x\to2} f(f(x))$? =

$f\left(\lim\limits_{x\to2} f(x)\right)$

$f(5)$

Since $5>2$

$8-\frac{3}{2}(5)$

$8-\frac{15}{2} = \frac{1}{2}$

(A) -15

(B) -7

(C) -1

(D) $\frac{1}{2}$

8.

The function h is defined by $h(x) = \frac{x^2-7}{x-3}$. Which of the following statements must be true?

@ 3 $\frac{2}{0}$

$3^- = \frac{\#}{-\text{small}} = -\infty$

$3^+ = \frac{\#}{+\text{small}} = +\infty$

(A) $\lim\limits_{x\to3^-} h(x) = -\infty$ and $\lim\limits_{x\to3^+} h(x) = -\infty$

(B) $\lim\limits_{x\to3^-} h(x) = +\infty$ and $\lim\limits_{x\to3^+} h(x) = -\infty$

(C) $\lim\limits_{x\to3^-} h(x) = -\infty$ and $\lim\limits_{x\to3^+} h(x) = +\infty$

(D) $\lim\limits_{x\to3^-} h(x) = +\infty$ and $\lim\limits_{x\to3^+} h(x) = +\infty$

9.

Let f be a function such that $\lim_{x \to 5^-} f(x) = \infty$. Which of the following statements must be true?

Ⓐ ~~$\lim_{x \to 5^+} f(x) = \infty$~~

Ⓑ ~~f is undefined at $x = 5$.~~

Ⓒ The graph of f has a vertical asymptote at $x = 5$.

Ⓓ The graph of f has a vertical asymptote at $x = -5$.

$$f(t) = \begin{cases} \overset{t^2+10t+25}{\cancel{\frac{t^3}{12}}} & \text{for } 0 \le t < 6 \\ g(t) & \text{for } 6 \le t \le 12 \end{cases}$$

t (hours)	6	8	10	12
$g(t)$ (cubic meters)	306	376	428	474

At an excavation site, the amount of dirt that has been removed, in cubic meters, is modeled by the function f defined above, where g is a differentiable function and t is measured in hours. Values of $g(t)$ at selected values of t are given in the table above.

10. (a) According to the model f, what is the average rate of change of the amount of dirt removed over the time interval $6 \le t \le 12$ hours?

$\frac{g(12)-g(6)}{12-6} = \frac{474-306}{12-6} = \frac{168}{6} = \boxed{28 \text{ m}^3/\text{h}}$

(b) Use the data in the table to approximate $f'(9)$, the instantaneous rate of change in the amount of dirt removed, in cubic meters per hour, at time $t = 9$ hours. Show the computations that lead to your answer.

$f'(9) \approx \frac{f(10)-f(8)}{10-8} = \frac{428-376}{2} = \frac{52}{2} = \boxed{26 \text{ m}^3/\text{h}}$

$f(2) = 96+4-\frac{8}{12} \approx 99\frac{1}{3} = \frac{298}{3}$

(d) Find $f'(2)$, the instantaneous rate of change in the amount of dirt removed, in cubic meters per hour, at time $t = 2$ hours.

need slope @ 2, $(2, 99)$ $(t, t^2+10t+25)$

$\lim_{t \to 2} \frac{t^2+10t+25-49}{t-2} = \frac{t^2+10t-24}{t-2}$

$\lim_{t \to 2} \frac{(t-2)(t+12)}{t-2} = \boxed{14 \text{ m}^3/\text{h}}$

x	-2	-1	0	1	2
$g(x)$	-3	2	1	0	5

11. Selected values of a function g are shown in the table above. What is the average rate of change of g over the interval $[-2, 2]$?

$\frac{g(2)-g(-2)}{2--2} = \frac{5--3}{4} = \frac{8}{4} = \boxed{2}$

Let f be the function defined by $f(x) = e^{2x}$. The average rate of change of f over the interval $[1, b]$ is 20, where $b > 1$. Which of the following is an equation that could be used to find the value of b?

$\frac{f(b)-f(1)}{b-1} = 20$

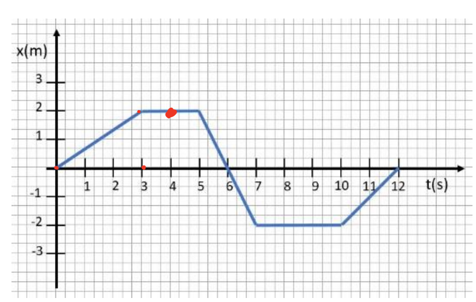
Ⓐ $f(b) = 20$

Ⓑ $f(b) - f(1) = 20$

Ⓒ $\frac{f(b)-f(1)}{b-1} = 20$

Ⓓ $\frac{f(b)+f(1)}{2} = 20$

12. The position of a particle is given by the following graph:



a) What is the speed of the particle at t = 4 seconds?

need slope $= \boxed{0}$

b) What is the speed of the particle at t = 2 seconds?

$m = \boxed{\frac{2}{3}}$

13. Suppose *f* is the function shown. Then find:

$$f(x) = \begin{cases} -x^2+3x+3 & \text{for} \quad x<2 \\ 6 & \text{for} \quad x=2 \\ 8-\frac{3}{2}x & \text{for} \quad x>2 \end{cases}$$

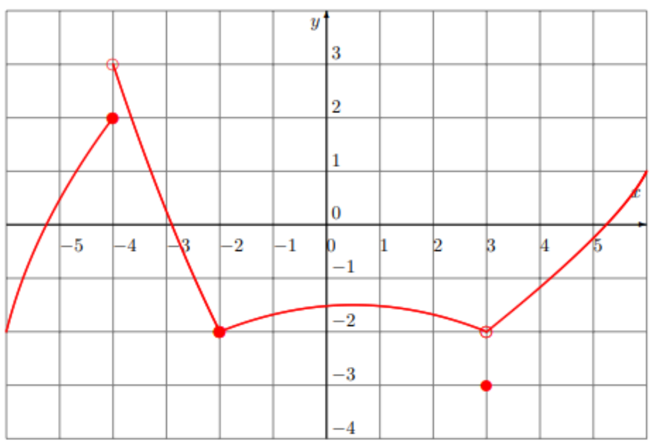
a) $\lim_{x\to 2^-} f(x) = -(2)^2+3(2)+3 = \boxed{5}$
$-4+6+3$

b) $\lim_{x\to 2^+} f(x) = 8-\frac{3}{2}(2) = \boxed{5}$

c) $\lim_{x\to 2} f(x)$ $\boxed{5}$

d) $f(2) = \boxed{6}$

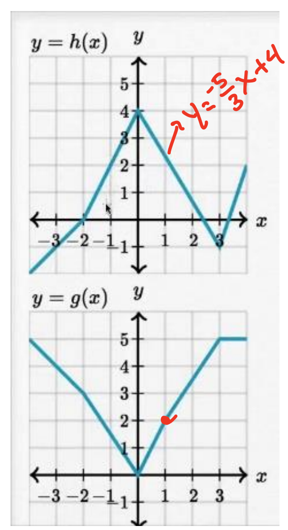
14. Suppose *f* is the function shown in the graph below:



Find the following limits:

a) $\lim_{x\to -2^-} f(x)$ $= \boxed{-2}$ b) $\lim_{x\to -2^+} f(x)$ $\boxed{-2}$ c) $\lim_{x\to -2} f(x)$ $\boxed{-2}$ d) $\lim_{x\to -4} f(x)$ $\boxed{\text{DNE}}$ e) $\lim_{x\to 3} f(x)$ $\boxed{-2}$

The graphs of *h*(x) and *g(x)* are shown. Solve each of the following.



a) $\lim_{x\to 0} \frac{h(x)}{g(x)} = \frac{\lim_{x\to 0} h(x)}{\lim_{x\to 0} g(x)} = \frac{4}{0} =$ DNE

b) $\lim_{x\to 1} \frac{h(x)}{g(x)} = \frac{2\frac{1}{3}}{2} = \boxed{\frac{7}{6}}$

$h(1) = -\frac{5}{3}+4$
$=$

c) $\lim_{x\to 3} \frac{h(x)+2}{g(x)} = \frac{1+2}{5} = \boxed{\frac{3}{5}}$

d) $\lim_{x\to -2} [3h(x)+2g(x)] =$ $3(0)+2(3) = \boxed{6}$

Let *f(x)* = 2x² – 1 and P be the point (-1, 1).

a) Find the slope of the ~~cure~~ curve *y = f(x)* at P.

$P(-1,1) \quad Q(x, 2x^2-1)$

$\lim_{x\to -1} \frac{2x^2-1-1}{x+1} = \frac{2x^2-2}{x+1} = \frac{2(x-1)\cancel{(x+1)}}{\cancel{(x+1)}} = 2(-1-1) = \boxed{-4 = m}$

b) The equation of the tangent at P.

$\boxed{y-1=-4(x+1)}$

c) The equation of the normal at P. $\rightarrow$ $m_\perp = \frac{1}{4}$

$m=-4$ so

$\boxed{y-1=\frac{1}{4}(x+1)}$