AP Calculus - Chapter 2 Test 1 - Review Problems

1.

x	3.9	3.99	3.999	3.9999	4.0001	4.001	4.01	4.1
$f(x)$	5	-25	125	-625	5.9999	5.999	5.99	5.9

The table above gives values of a function f at selected values of x. Which of the following conclusions is supported by the data in the table?

Ⓐ $\lim_{x \to 4} f(x) = 6$

Ⓑ $\lim_{x \to 4^-} f(x) = 6$

Ⓒ $\lim_{x \to 4^+} f(x) = 6$

Ⓓ $\lim_{x \to 6^+} f(x) = 4$

2.

If f is the function defined by $f(x) = \frac{x-9}{\sqrt{x}-3}$, then $\lim_{x \to 9} f(x)$ is equivalent to which of the following?

Ⓐ $\lim_{x \to 9} (\sqrt{x} - 3)$

Ⓑ $\lim_{x \to 9} (\sqrt{x} + 3)$

Ⓒ $\lim_{x \to 9} \left(\frac{x^2-81}{x-9}\right)$

Ⓓ $\frac{\lim_{x \to 9}(x-9)}{\lim_{x \to 9}(\sqrt{x}-3)}$

3.Find each limit:

$\lim_{x \to \infty} \frac{20}{10+e^{-x}}=$ $\lim_{x \to 0} \frac{cosx}{sinx-e^{x}}=$ $\lim_{x \to 0} \frac{7x^5+5x^2+12x}{3x^5+4x}$ is

4.

If $f(x) = \frac{\sin x - 1}{\cos^2 x}$, then $\lim_{x \to \frac{\pi}{2}} f(x)$ is equivalent to which of the following?

Ⓐ $\lim_{x \to \frac{\pi}{2}} \frac{-1}{1+\sin x}$

Ⓑ $\lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{1+\sin^2 x}$

Ⓒ $\lim_{x \to \frac{\pi}{2}} \sec x$

Ⓓ $\lim_{x \to \frac{\pi}{2}} (\tan x - \sec x)$

5.

If f is the function defined by $f(x) = \frac{\frac{1}{x}-1}{x-1}$, then $\lim_{x\to 1} f(x)$ is equivalent to which of the following?

Ⓐ $\lim_{x\to 1}\left(-\frac{1}{x}\right)$

Ⓑ $\lim_{x\to 1}\left(\frac{1}{x^2}-1\right)$

Ⓒ $\lim_{x\to 1}\left(\frac{x-1}{x-1}\right)$

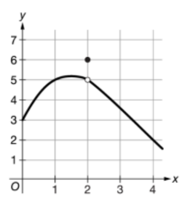
Ⓓ $\frac{\lim_{x\to 1}\left(\frac{1}{x}-1\right)}{\lim_{x\to 1}(x-1)}$

6.

The function g is given by $g(x) = \frac{7x-26}{x-5}$. The function h is given by $h(x) = \frac{3x+14}{2x+1}$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for $0 < x < 5$, what is $\lim_{x\to 2} f(x)$?

7.

$$f(x) = \begin{cases} -x^2+3x+3 & \text{for} \quad x < 2 \\ 6 & \text{for} \quad x = 2 \\ 8-\frac{3}{2}x & \text{for} \quad x > 2 \end{cases}$$



Let f be the piecewise function defined above. Also shown is a portion of the graph of f. What is the value of $\lim_{x\to 2} f(f(x))$?

Ⓐ -15

Ⓑ -7

Ⓒ -1

Ⓓ $\frac{1}{2}$

8.

The function h is defined by $h(x) = \frac{x^2-7}{x-3}$. Which of the following statements must be true?

Ⓐ $\lim_{x\to 3^-} h(x) = -\infty$ and $\lim_{x\to 3^+} h(x) = -\infty$

Ⓑ $\lim_{x\to 3^-} h(x) = +\infty$ and $\lim_{x\to 3^+} h(x) = -\infty$

Ⓒ $\lim_{x\to 3^-} h(x) = -\infty$ and $\lim_{x\to 3^+} h(x) = +\infty$

Ⓓ $\lim_{x\to 3^-} h(x) = +\infty$ and $\lim_{x\to 3^+} h(x) = +\infty$

9.

Let f be a function such that $\lim_{x \to 5^-} f(x) = \infty$. Which of the following statements must be true?

Ⓐ $\lim_{x \to 5^+} f(x) = \infty$

Ⓑ f is undefined at $x = 5$.

Ⓒ The graph of f has a vertical asymptote at $x = 5$.

Ⓓ The graph of f has a vertical asymptote at $x = -5$.

$$f(t) = \begin{cases} t^2 + 10t + 25 & 0 \le t < 6 \\ g(t) & \text{for } 6 \le t \le 12 \end{cases}$$

t (hours)	6	8	10	12
$g(t)$ (cubic meters)	306	376	428	474

At an excavation site, the amount of dirt that has been removed, in cubic meters, is modeled by the function f defined above, where g is a differentiable function and t is measured in hours. Values of $g(t)$ at selected values of t are given in the table above.

10. (a) According to the model f, what is the average rate of change of the amount of dirt removed over the time interval $6 \le t \le 12$ hours?

(b) Use the data in the table to approximate $f'(9)$, the instantaneous rate of change in the amount of dirt removed, in cubic meters per hour, at time $t = 9$ hours. Show the computations that lead to your answer.

(d) Find $f'(2)$, the instantaneous rate of change in the amount of dirt removed, in cubic meters per hour, at time $t = 2$ hours.

x	-2	-1	0	1	2
$g(x)$	-3	2	1	0	5

11. Selected values of a function g are shown in the table above. What is the average rate of change of g over the interval $[-2, 2]$?

Let f be the function defined by $f(x) = e^{2x}$. The average rate of change of f over the interval $[1, b]$ is 20, where $b > 1$. Which of the following is an equation that could be used to find the value of b?

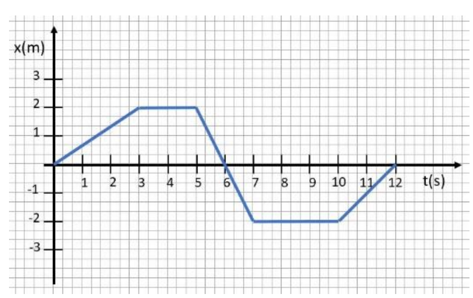
Ⓐ $f(b) = 20$

Ⓑ $f(b) - f(1) = 20$

Ⓒ $\frac{f(b) - f(1)}{b - 1} = 20$

Ⓓ $\frac{f(b) + f(1)}{2} = 20$

12. The position of a particle is given by the following graph:



a) What is the speed of the particle at t = 4 seconds?

b) What is the speed of the particle at t = 2 seconds?

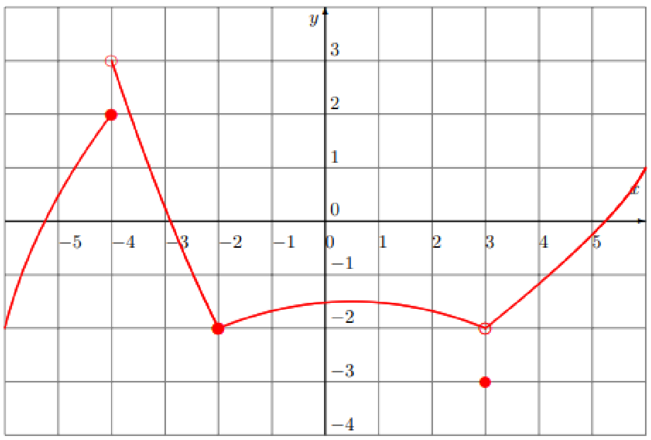
13. Suppose *f* is the function shown. Then find:

$$f(x) = \begin{cases} -x^2+3x+3 & \text{for} \quad x<2 \\ 6 & \text{for} \quad x=2 \\ 8-\frac{3}{2}x & \text{for} \quad x>2 \end{cases}$$

a) $\lim_{x\to 2^-} f(x)$ b) $\lim_{x\to 2^+} f(x)$

c) $\lim_{x\to 2} f(x)$ d) $f(2)$

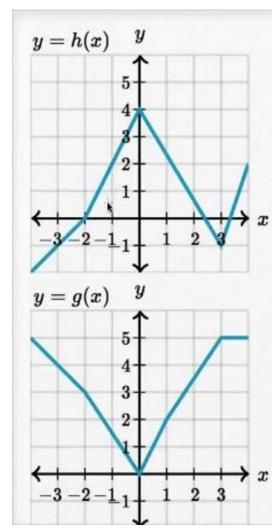
14. Suppose *f* is the function shown in the graph below:



Find the following limits:

a) $\lim_{x\to -2^-} f(x)$ b) $\lim_{x\to -2^+} f(x)$ c) $\lim_{x\to -2} f(x)$ d) $\lim_{x\to -4} f(x)$ e) $\lim_{x\to 3} f(x)$

The graphs of *h*(x) and *g(x)* are shown. Solve each of the following.



a) $\lim_{x\to 0} \frac{h(x)}{g(x)} =$ b) $\lim_{x\to 1} \frac{h(x)}{g(x)} =$

c) $\lim_{x\to 3} \frac{h(x)+2}{g(x)} =$ d) $\lim_{x\to -2} [3h(x)+2g(x)] =$

Let *f(x)* = $2x^2 - 1$ and P be the point (-1, 1).

a) Find the slope of the cure *y = f(x)* at P.

b) The equation of the tangent at P.

c) The equation of the normal at P.