AP Calculus Semester 1 Final Review 2

NO Calculator

Multiple Choice

1. If $y = (x^3+1)^2$, then $\frac{dy}{dx} =$

(A) $2(x^3+1)$ **(B)** $2(3x^2+1)$ **(C)** $3x^2(x^3+1)$ **(D)** $6x^2(x^3+1)$

2. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) $0.04\pi\ m^2/\sec$ **(B)** $0.4\pi\ m^2/\sec$ **(C)** $4\pi\ m^2/\sec$ **(D)** $20\pi\ m^2/\sec$

3. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

(A) $-\frac{7}{2}$ **(B)** -2 **(C)** $\frac{2}{7}$ **(D)** $\frac{3}{2}$

4. $\lim\limits_{x\to 0} \frac{\sin x}{x^2+3x}$ is

(A) 1 **(B)** $\frac{1}{3}$ **(C)** ∞ **(D)** 3

5. If $f(x) = \sin(e^{-x})$, then $f'(x) =$

(A) $e^{-x}\cos(e^{-x})$ **(B)** $\cos(e^{-x}) + e^{-x}$ **(C)** $\cos(e^{-x}) - e^{-x}$ **(D)** $-e^{-x}\cos(e^{-x})$

6. The slope of the tangent to the curve $xy^3 + x^2y^2 = 6$ at $(2,1)$ **is**

(A) $-\frac{3}{2}$ **(B)** -1 **(C)** $-\frac{5}{14}$ **(D)** $-\frac{3}{14}$

7. A point moves along the curve y = x^2 + 1 in such a way that when x = 4, the x-coordinate is increasing at a rate of 5 feet per second. At what rate is the y-coordinate changing at that time?

(A) $80\ ft/\sec$ **(B)** $45\ ft/\sec$ **(C)** $85\ ft/\sec$ **(D)** $40\ ft/\sec$

The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

8.

(A) $x < \frac{2}{3}$ or $x > 2$ **(B)** $\frac{2}{3} < x < 2$ **(C)** $x < 2$ or $x > 6$ **(D)** $2 < x < 6$

$\lim_{x \to 0} \frac{\sin 3x}{\cos 2x - x - 1}$ is

9.

(A) 0 **(B)** -3 **(C)** -1 **(D)** does not exist

10.

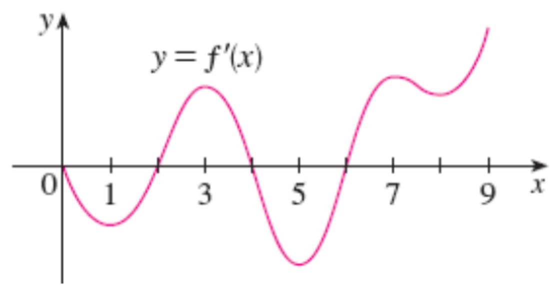
The graph of $y = f'(x)$ is given to the right. When is $f(x)$ concave up from $[0,6]$?

(A) $(1,3) \cup (5,6)$

(B) $(0,2) \cup (5,6)$

(C) $(2,4)$

(D) $(0,2) \cup (4,6)$



$\lim_{h \to 0} \frac{\sqrt[3]{8+h} - 2}{h}$ is

11.

(A) $\frac{1}{12}$ **(B)** 2 **(C)** $\frac{1}{4\sqrt{2}}$ **(D)** $\frac{1}{6}$

x	f	f'	g	g'
0	2	1	5	-4
1	3	2	3	-3
2	5	3	1	-2
3	10	4	0	-1

If $q(x) = g(f(x))$, find the value of $q'(1)$.

12.

(A) -2 **(B)** -1 **(C)** -12 **(D)** 4

If $k(x) = 5xg(x)$, find the value of $k'(2)$.

13.

(A) -10 **(B)** 5 **(C)** -15 **(D)** 25

14.

Which of the following are true of the graph to the right?

I. $\lim_{x \to -2} f(x) = 5$

II. $\lim_{x \to 1} f(x)$ exists

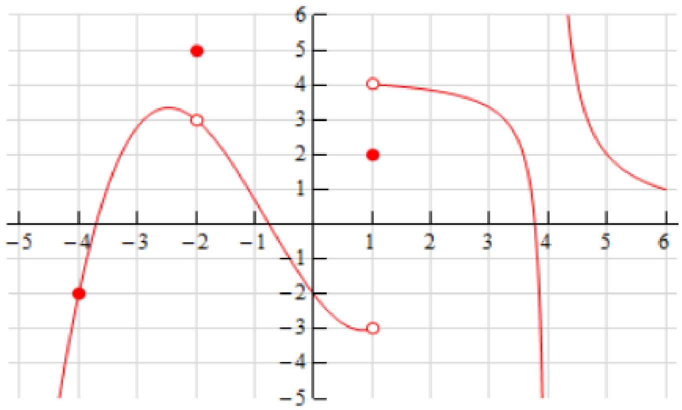
III. f is continuous at $x = -4$

(A) I & III

(B) I, II, & III

(C) III only

(D) II & III



$$\lim_{x \to -\infty} \frac{\sqrt{5x^2 + 3}}{13x^2 + 2} \text{ is}$$

15.

(A) $-\frac{\sqrt{5}}{13}$ **(B)** $\frac{\sqrt{5}}{13}$ **(C)** 0 **(D)** non existant

If $f(x) = -x^3 + x^2 + x - 3$ find when $f(x)$ is concave up.

16.

(A) $x < \frac{1}{3}$ **(B)** $x > \frac{1}{3}$ **(C)** $x < -\frac{1}{3}$ **(D)** $x > -\frac{1}{3}$

17.

If $y' = x^4 - 4x^3 - 2$ find the x-coordinate(s) in which y goes from concave up to concave down.

(A) 0 & 3 **(B)** 0 **(C)** 3 **(D)** none

18.

. If $f(x)$ is a twice differentiable function, which of the values for $f'(c)$ would guarantee a value for c in which the average rate of change is equal to the instantaneous rate of change on the interval $2 \le x \le 3$?

(A) 2 **(B)** 5 **(C)** 10 **(D)** 1

19. Which of the following are true for the function $f(x) = -(x-2)^{2/3}$?

I. f is continous at $x = 2$

II. f is differentiable at $x = 2$

III. f has an absolute maximum at $x = 2$

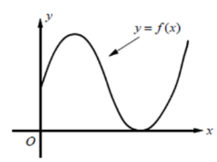
(A) I & III **(B)** I, II, & III **(C)** I only **(D)** I & II **(D)** II & III

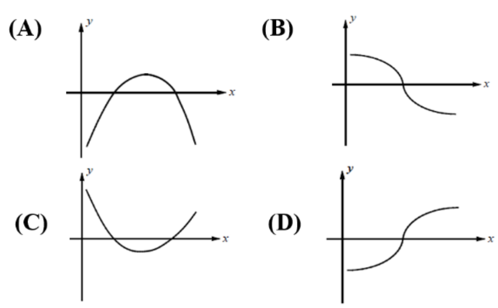
20. The line $4x + y = k$ is tangent to the graph of $y = 2x^2 - 8x + 9$. What is the value of k?

A) -9 B) 0 C)4 D) 7 E) 15

Calc Required

1. The graph of f is shown in the figure to the right. Which of the following could be f'?





2. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \ge 3 \end{cases}$ is

(A) undefined.

(B) continuous but not differentiable.

(C) differentiable but not continuous.

(D) neither continuous nor differentiable.

(E) both continuous and differentiable.

3. The radius of a sphere is decreasing at a rate of 2 cm/sec. At the instant when the radius of the sphere is 3 cm, what is the rate of change, in square meters per second, of the surface area of the sphere? ($S = 4\pi r^2$)

(A) -108π **(B)** -72π **(C)** -48π **(D)** -24π

4.

The first derivative of the function f is defined by $f'(x) = \cos(x^3 - x)$ for $0 \le x \le 2$. On what intervals is f decreasing?

(A) $0 < x < 0.577$ and $1 < x < 1.691$

(B) $1.445 < x < 1.875$

(C) $1 < x < 1.445$ and $1.875 < x < 2$

(D) $0 < x < 1.445$ and $1.875 < x < 2$

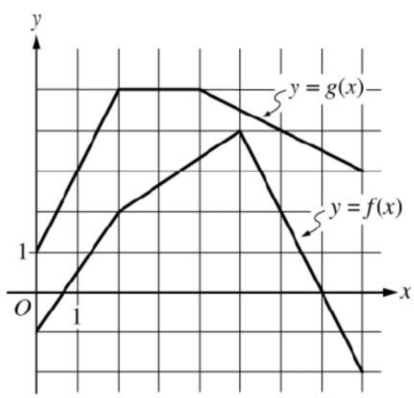
5. Use the table to find:

$h'(3)$ when $h(x) = \dfrac{f(x)}{g(x)}$.

(A) -2
(B) 1.5
(C) 3
(D) 5

$f(3)$	$g(3)$	$f'(3)$	$g'(3)$
-1	2	4	-2

6.



The graphs of f and g are shown above. If $h(x) = f(x)g(x)$, then $h'(6) =$

(A) -9 (B) -7 (C) 1 (D) 7 (E) 9

7.

In the xy-plane, the graph of the twice-differentiable function $y = f(x)$ is concave up on the open interval $(0, 2)$ and is tangent to the line $y = 3x - 2$ at $x = 1$. Which of the following statements must be true about the derivative of f?

(A) $f'(x) \le 3$ on the interval $(0.9, 1)$.

(B) $f'(x) \ge 3$ on the interval $(0.9, 1)$.

(C) $f'(x) < 0$ on the interval $(0.9, 1.1)$.

(D) $f'(x) > 0$ on the interval $(0.9, 1.1)$.

(E) $f'(x)$ is constant on the interval $(0.9, 1.1)$.

Free Response No Calc

1.

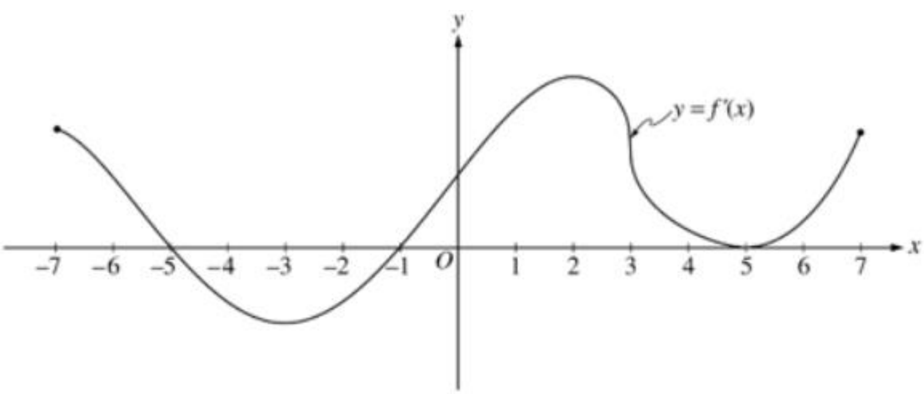
Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

2.



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

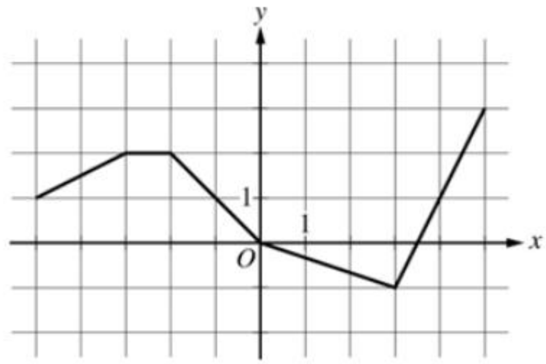
(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

x	$g(x)$	$g'(x)$
−5	10	−3
−4	5	−1
−3	2	4
−2	3	1
−1	1	−2
0	0	−3



Graph of h

. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

4.

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

(a) Find $f'(x)$ and $f''(x)$.

(b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

(c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

Free Response Calc Allowed

A company produces and sells chili powder. The company's weekly profit on the sale of x kilograms of chili powder is modeled by the function P given by $P(x) = 48x + 1.4x^2 - 0.05x^{2.8} - 270$, where $P(x)$ is in dollars and $0 \le x \le 80$.

(a) Find the rate, in dollars per kilogram, at which the company's weekly profit is changing when it sells 32 kilograms of chili powder. Is the company's weekly profit increasing or decreasing when it sells 32 kilograms of chili powder? Give a reason for your answer.

(b) How many kilograms of chili powder must the company sell to maximize its weekly profit? Justify your answer.

1.

2.

For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

Answers:

MC

1. D 2. C 3. A 4. B 5. D 6. C 7. D 8. B 9. B 10. A 11. A 12. A 13. C

14. C 15. C 16. A 17. D 18. B 19. A 20. D

Calc

1. C 2. B 3. C 4. B 5. B 6. A 7. A

Free Response No Calc

1. A) take the derivative and show it

B) (3, 2) C) -14/49, maximum

2. a) x = -1 only because f' changes from – to +

b) x = -5 only because f' changes from + to –

c) $f''(x)$ exists and f' is decreasing on the intervals $(-7,-3)$, $(2,3)$, and $(3,5)$

d) x = 7

The absolute maximum must occur at $x = -5$ or at an endpoint.
$f(-5) > f(-7)$ because f is increasing on $(-7,-5)$

The graph of f' shows that the magnitude of the negative change in f from $x = -5$ to $x = -1$ is smaller than the positive change in f from $x = -1$ to $x = 7$. Therefore the net change in f is positive from $x = -5$ to $x = 7$, and $f(7) > f(-5)$. So $f(7)$ is the absolute maximum.

3. a) -1 b) 1/3 c) -3

d) Yes by MVT – g is differentiable and (2-10)/(-3+5) = -8/2 = -4

4. a) $f'(x) = \frac{1}{2} kx^{-1/2} - 1/x$ $f''(x) = -\frac{1}{4} kx^{-3/2} + x^{-2}$

b) k = 2, relative min

At this inflection point, $f''(x) = 0$ and $f(x) = 0$.

$$f''(x) = 0 \Rightarrow \frac{-k}{4x^{3/2}} + \frac{1}{x^2} = 0 \Rightarrow k = \frac{4}{\sqrt{x}}$$
$$f(x) = 0 \Rightarrow k\sqrt{x} - \ln x = 0 \Rightarrow k = \frac{\ln x}{\sqrt{x}}$$

Therefore, $\frac{4}{\sqrt{x}} = \frac{\ln x}{\sqrt{x}}$

$\Rightarrow 4 = \ln x$

$\Rightarrow x = e^4$

$\Rightarrow k = \frac{4}{e^2}$

c)

Calc Allowed

(a) $P'(32) = 65.920$

Since $P'(32) > 0$, the company's profit is increasing when it sells 32 kilograms of chili powder.

(b) $P'(x) = 48 + 2.8x - 0.14x^{1.8} = 0$ when $x = 58.358152$

x	$P(x)$
0	−270
58.358152	2893.04
80	1873.32

The company must sell 58.358 kilograms of chili powder to maximize its profit.

1.

(a) $v(4) = 2.978716 > 0$
$v'(4) = -1.164000 < 0$

The particle is slowing down since the velocity and acceleration have different signs.

(b) $v(t) = 0 \Rightarrow t = 2.707468$

$v(t)$ changes from positive to negative at $t = 2.707$. Therefore, the particle changes direction at this time.

2.