

AP CALCULUS TEST CHECKLIST

WHAT TO BRING

- Student ID
- Graphing Calculator - MAKE SURE YOU CHARGE IT/Bring extra batteries
- Several No. 2 Sharpened pencils
- Pen (blue or black ink) to sign or fill out certain forms
- (Optional) – Snacks for breaks
- (Optional) – Water bottle but must be in a clear plastic bottle with label removed

DO NOT BRING

- Hydroflasks, etc... only a clear plastic water bottle is allowed with label removed
- Cell phones or electronic devices
- Study materials
- Timing devices
- Do not wear clothing with formulas, etc.
- Leave your backpack in your car or with a trusted teacher
- Timing devices (you can wear a watch without any beeps, etc)
- APPLE/SMARTWATCHES

LAST MIN CONCEPTS TO REVIEW

*Review the 6 common FRQ Questions

- Accumulation (rates in, out)
- Particle Motion
- Area/Volume
- Riemann Sum
- FTC/Derivative Graph
- Differential Equations/Separation of Variables

Can you answer each of these:

- 1) Find the equation of the tangent to $f(x)$ at (a, b)
- 2) If you are given a table of x and $f(x)$ on selected values between a and b , how do you estimate the rate $f'(c)$ if c is between a and b ?
- 3) How do you show a piecewise function is differentiable at a point a ?
- 4) How do you find the inverse derivative of a function?
- 5) How do you find the average value of a function?
- 6) When you look at a **derivative** graph:
 - a) What do the horizontal tangents tell you?
 - b) What do the x -intercepts tell you?

- c) What does a (+) or (-) slope tell you?
- d) What does it mean when the graph is above or below the axis?
- e) What does the area under the graph tell you?

7) $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

8) $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

9) $\lim_{x \rightarrow 0} \sin \frac{1}{x}$

10) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$

- 11) Defn of Continuity at a point
- 12) What is IVT?
- 13) Extreme Value Theorem
- 14) L'Hopital's Rule?? When is it used and how?
- 15) What is the definition of a derivative?
- 16) If a function is continuous, is it differentiable? If a function is differentiable is it continuous?
- 17) What is Rolle's Theorem
- 18) What is MVT?
- 19) What is MVT geometrically?
- 20) What is the product rule?
- 21) What is the quotient rule?
- 22) What is the chain rule? How would you find the derivative of $f(g(x))$?
- 23) How do you differentiate implicitly?
- 24) know the derivative of sin, cos, tan, sec, csc, cot, arcsin, arcos, arctan, arcsec, arccsc, arccot
- 25) What is derivative of e^x ? Suppose you had e^{2x} ?
- 26) What is the derivative of a^x ?
- 27) What is the derivative of $\ln x$? $\ln x^2$
- 37) How do you find the average rate of change of $f(x)$ on an interval $[a, b]$?
- 38) How do you find the INSTANTANEOUS rate of change of $f(x)$ at $x = a$?
- 39) How do you find critical points?
- 40) How do you find intervals of increasing/decreasing?
- 41) How do you find intervals of concavity?
- 42) How do you find maximums and minimums? Absolute max/min?
- 43) How do you find inflection points?
- 44) How are position, velocity and acceleration related?
- 45) How do you find the slope of a tangent at a point?
- 46) How do you know if speed is increasing or decreasing i.e. speeding up or slowing down?
- 47) What is the linear approximation/linearization formula?
- 48) What is a normal line?
- 49) How do you find Riemann Summs?
- 50) What is the FTC?
- 51) How can you rearrange
- 52) Find $\frac{d}{dx} \int_a^{g(x)} f(t) dt$
- 53) How do you apply the trapezoid rule?
- 54) How do you know if the area is an over or underestimate?

55) Find $\int u^n du$

56) Know the integrals of trig functions, sin, cos, tan, etc...

57) $\int \frac{du}{\sqrt{1-u^2}}$

58) $\int \frac{du}{1+u^2}$

59) $\int \frac{du}{u\sqrt{u^2-1}}$

60) $\int e^u du$

61) $\int a^u du$

62) $\int \frac{du}{u}$

63) How do you find displacement?

64) How do you find distance travelled?

65) How do you find the area between curves f and g on [a, b]?

66) If R(t) represents a rate of change, then $\int_a^b f(t)dt$ represents what?

67) How do you find volume by disks?

68) How do you find volume by washers?

69) How do you find cross sectional area?

70) How do you find the average value of a function on [a, b]?

71) What is the MVT for integrals?

72) What are the steps to separating variables?

Answers:

1. Find the derivative $f'(a) = m$, then use $y - y_1 = m(x - x_1)$ where (x_1, y_1) is (a, b) .

2. Straddle c by using values close to it and find the slope. So suppose k is greater and h is smaller, then

$$f'(c) = \frac{f(k) - f(h)}{k - h}$$

3) take the lim from left and right to verify continuity... if they are =, then it is continuous. Now the derivative from the left must equal the derivative from the right.

4) $(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$

5) $\frac{1}{b-a} \int_a^b f(x)dx$

6) a) Zero's for $f''(x)$

b) Zero's for $f'(x)$

c) Increasing or decreasing $f''(x)$

d) where f' is (+) or (-), thus if the original function is increasing or decreasing

e) the value of the integral $\int_a^b f(x)dx$

7) 1 8) 0 9) DNE 10) 0

11) i) $f(a)$ exists ii) $\lim_{x \rightarrow a} f(x)$ exists and iii) $\lim_{x \rightarrow a} f(x) = f(a)$

12) If f is continuous on [a, b] and if k is between f(a) and f(b), then there exists a c in [a, b] such that $f(c) = k$ Applies to a point between 2 points on a continuous function

13) If f is continuous on [a, b], then f has a max and a min on [a, b]

14) When finding limits, If indeterminate, then take derivative of top and bottom to find limit

$$15) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

*Know how to identify the equation and just take the derivative at the point given

16) can be continuous but not differentiable (a kink)

17) If f is continuous on $[a, b]$ and differentiable on (a, b) and $f(a) = f(b)$,

then for some c in (a, b) $f'(c) = 0$. Similar to MVT, but on a flat/level surface

18) If f is continuous on $[a, b]$ and differentiable on (a, b) , then for some c in (a, b)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

19) If f is continuous on $[a, b]$ and differentiable on (a, b) , then the **line tangent** to the curve at some point in the interval is **parallel to the secant** line across the interval.

20) $f' s' + s f'$

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2} = \frac{BT' - TB'}{B^2}$$

21)

22) $d/dx f(g(x)) = f'(g(x)) \cdot g'(x)$ (take the derivative of the outside, then the inner parts)

23) Take the derivative of all the x and y 's, each time you take a derivative of y you need to put in a y' or dy/dx , then solve for the y' (dy/dx)

Derivatives of Trigonometric Functions

$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\csc x) = -\csc x \cot x$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\sec x) = \sec x \tan x$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\cot x) = -\csc^2 x$

24)

25) $e^x, 2e^{2x}$ apply chain rule!

26) $a^x \ln a$ *don't forget to apply chain rule!

27) $1/x, 1/x^2 * 2x = 2/x$ *don't forget the chain rule!

37) Slope!! $\frac{f(b) - f(a)}{b - a}$

38) $f'(a)$

39) set $f' = 0$, test endpoints and where f' does not exist

40) use critical points of f' on a number line to find +++ and ---- for increasing/decreasing

41) set $f''(x) = 0$ and solve... use those points on a number line to find +++ and --- for concave up/down.

42) Get the critical points by setting $f'(x) = 0$ OR UNDEFINED, put them on a number line of $f'(x)$ to determine +++ and ---- If changes from (+) to (-) then rel max, if (-) to (+) then rel min. For absolute make sure to test the endpoints!!

43) Set $f''(x) = 0$ OR UNDEFINED, yields possible inflection points... put them on a f'' number line to determine +++ and ---, where it changes is and inflection point

44) derivatives or integrals of each other. Since $x'(t) = v(t)$ and $v'(t) = a(t)$, $\int a(t) dt = v(t)$ and $\int v(t) dt = x(t)$

45) Find the derivative at that point.

Derivatives of Inverse Trigonometric functions

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \cdot \frac{du}{dx}$	$\frac{d}{dx} \csc^{-1} x = \frac{-1}{ u \sqrt{u^2-1}} \cdot \frac{du}{dx}$
$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \cdot \frac{du}{dx}$	$\frac{d}{dx} \sec^{-1} x = \frac{1}{ u \sqrt{u^2-1}} \cdot \frac{du}{dx}$
$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2} \cdot \frac{du}{dx}$	$\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2} \cdot \frac{du}{dx}$

- 46) Increasing, both $v(t)$ and $a(t)$ same sign... decreasing is opposite signs
 47) $f(x) \approx f(a) + f'(a)(x - a)$
 48) Perpendicular to a tangent line
 49) Set up according to left/right/midpoint traps to find the area under the curve

50) i) $\int_a^b f(x) dx = F(b) - F(a)$ *helps us to find area under the curve

ii) $F(x) = \int_a^x f(t) dt$ so $F'(x) = f(x)$ i.e. the derivative of an integral is the function itself... but APPLY THE CHAIN RULE!

51) $-\int_a^b f(x) dx$

52) $f(g(x)) \cdot g'(x)$

53) $h/2 (y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$ *remember that $h/2$ and only middle terms have 2 times the y -value

54) If Riemann Sum rectangles are above or below the curve **But you must also examine if the curve is increasing or decreasing

55) $\frac{u^{n+1}}{n+1} + C$

Basic Trig Derivatives and their Corresponding Indefinite Integrals	
$\frac{d}{dx} \sin x = \cos x$	$\int \cos x dx = \sin x + C$
$\frac{d}{dx} \cos x = -\sin x$	$\int \sin x dx = -\cos x + C$
$\frac{d}{dx} \tan x = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx} \sec x = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + C$
$\frac{d}{dx} \csc x = -\csc x \cot x$	$\int \csc x \cot x dx = -\csc x + C$
$\frac{d}{dx} \cot x = -\csc^2 x$	$\int \csc^2 x dx = -\cot x + C$

56)

57) $\arcsin u + C$

58) $\arctan u + C$

60) $e^u + C$

61) $\frac{a^u}{\ln a} + C$

*note $\int \tan x = -\ln|\cos x| + C$

59) $\operatorname{arcsec}|u| + C$

62) $\ln|u| + C$

63) $\int_a^b v(t) dt$

64) $\int_a^b |v(t)| dt$

65) $\int_a^b (f(x) - g(x)) dx$

66) the accumulated amount of change during the time interval from $a \leq t \leq b$

67)

HORIZONTAL AXIS OF REVOLUTION	VERTICAL AXIS OF REVOLUTION
$V = \pi \int_a^b [R(x)]^2 dx$	$V = \pi \int_a^b [R(y)]^2 dy$

***If moved off of x- or y-axis, then radius = equation – line of rotation

Notice how our rectangular slice, dx or dy, is perpendicular to the axis of revolution

***If moved off of x- or y-axis \rightarrow Radius = equation – line of rotation

68) $V = \pi \int_a^b [(R(x))^2 - (r(x))^2] dx$

69) Know the basic shape of the cross section – square, rectangle, semi-circle, triangle.

**Remember that \rightarrow Volume = \int Area*

Find the area equation and integrate!

70) $\frac{1}{b-a} \int_a^b f(x) dx$

71) If f is continuous on $[a, b]$, then there exists a c in $[a, b]$ such that $(b - a)f(c) = \int_a^b f(x) dx$

72) Separate the y and dy , x and dx

- Integrate each side and put + C on one side only
- Plug in initial condition BEFORE you solve for y !!! Much easier to get C this way and makes the problem easier. Also on FRQ, they will want that answer for C
- Find the particular solution by solving for y
- You will often have integrals that involve \ln or e , make sure to review the integral of $1/x$ and substitution rule for this