Chapter 3 Derivatives and Things to Memorize

Power Rule

➔ $\frac{d}{dx}(x^n) = nx^{n-1}$

Derivative of e

$\frac{d}{dx}(e^x) = e^x$ → with chain rule $\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$

*Remember the product rule as:

First·Derivative of Second + Second·Derivative of First

or F(x) = f(x) · g(x)

F '(x) = f(x) · g '(x) + g(x) · f '(x)

QUOTIENT RULE

If f & g are both differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$\frac{B \cdot T' - T \cdot B'}{B^2}$$

Or Remember it as:

$$\frac{\text{Bottom} \cdot \text{Derivative of Top} - \text{Top} \cdot \text{Derivative of Bottom}}{\text{Bottom Squared}}$$

SUMMARY

$\frac{d}{dx}(c) = 0$ $\qquad$ $\frac{d}{dx}(x^n) = nx^{n-1}$

$(cf') = cf'$ $\qquad$ $(f+g)' = f' + g'$ $\qquad$ $(f-g)' = f' - g'$

$(f \cdot g)' = f \cdot g' + g \cdot f'$ $\qquad$ $\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

<u>OTHER DERIVATIVES</u>

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

<u>Chain Rule Basics</u>

Basically → take the derivative of the outside • derivative of the inside

Thus $f(x) = (2x^2 + 3)^2$

$f'(x) = 2(2x^2 + 3) \bullet 4x$

$= 8x(2x^2 + 3)$

So when combined with power rule:

If n is any real number and u = g(x) is differentiable then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

or

$$*\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$$

<u>Logarithms</u>

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

with chain rule $\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$

Also,

$$\frac{d}{dx}(\ln g(x)) = \frac{g'(x)}{g(x)}$$

More Rules:

$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$ with chain rule $\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a} \cdot \frac{du}{dx}$

$\frac{d}{dx}(a^x) = a^x \ln a$ with chain rule $\frac{d}{dx}(a^u) = a^u \ln a \cdot \frac{du}{dx}$

Inverse Trig Functions

Formulas for Derivative of Inverse Trig Functions:

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ with chain rule $\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$

$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$ with chain rule $\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ with chain rule $\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$

$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$ with chain rule $\frac{d}{dx}(\csc^{-1} u) = -\frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$ with chain rule $\frac{d}{dx}(\sec^{-1} u) = \frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx}$

$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$ with chain rule $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+u^2} \cdot \frac{du}{dx}$

Newton's Law of Cooling

$$T - T_s = (T_o - T_s)e^{kt}$$

Linearization

$$f(x) \approx f(a) + f'(a)(x - a) = L(x)$$