

Chapter 3 Derivatives and Things to Memorize

Power Rule

$$\rightarrow \frac{d}{dx}(x^n) = nx^{n-1}$$

Derivative of e

$$\frac{d}{dx}(e^x) = e^x \rightarrow \text{with chain rule } \frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$$

*Remember the product rule as:

First·Derivative of Second + Second·Derivative of First

$$\text{or } F(x) = f(x) \cdot g(x)$$

$$F'(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

QUOTIENT RULE

If f & g are both differentiable, then

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \cdot \frac{d}{dx}[f(x)] - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$\frac{B T' - T B'}{B^2}$$

Or Remember it as:

$$\frac{\text{Bottom} \cdot \text{Derivative of Top} - \text{Top} \cdot \text{Derivative of Bottom}}{\text{Bottom Squared}}$$

SUMMARY

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$(cf)' = cf'$$

$$(f + g)' = f' + g'$$

$$(f - g)' = f' - g'$$

$$(f \cdot g)' = f \cdot g' + g \cdot f'$$

$$\left(\frac{f}{g} \right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

OTHER DERIVATIVES

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

Chain Rule Basics

Basically → take the derivative of the outside • derivative of the inside

$$\text{Thus } f(x) = (2x^2 + 3)^2$$

$$f'(x) = 2(2x^2 + 3) \cdot 4x$$

$$= 8x(2x^2 + 3)$$

So when combined with power rule:

If n is any real number and $u = g(x)$ is differentiable then

$$\frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$$

or

$$* \frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$$

Logarithms

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\text{with chain rule } \frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$$

Also,

$$\frac{d}{dx}(\ln g(x)) = \frac{g'(x)}{g(x)}$$

More Rules:

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\text{with chain rule } \frac{d}{dx}(\log_a u) = \frac{1}{u \ln a} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\text{with chain rule } \frac{d}{dx}(a^u) = a^u \ln a \cdot \frac{du}{dx}$$

Inverse Trig Functions

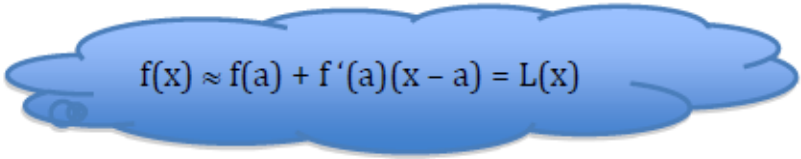
Formulas for Derivative of Inverse Trig Functions:

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	with chain rule $\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$
$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$	with chain rule $\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	with chain rule $\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$
$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$	with chain rule $\frac{d}{dx}(\csc^{-1} u) = -\frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx}$
$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$	with chain rule $\frac{d}{dx}(\sec^{-1} u) = \frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx}$
$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$	with chain rule $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+u^2} \cdot \frac{du}{dx}$

Newton's Law of Cooling

$$T - T_\varepsilon = (T_o - T_\varepsilon)e^{kt}$$

Linearization


$$f(x) \approx f(a) + f'(a)(x - a) = L(x)$$