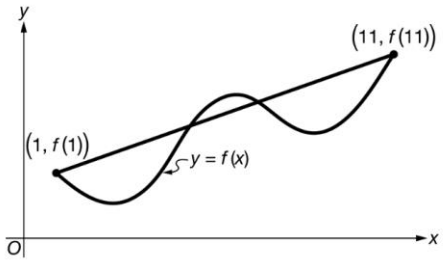
# AP Calculus Semester 1 Final – Additional MC Practice Problems

No calculator allowed.

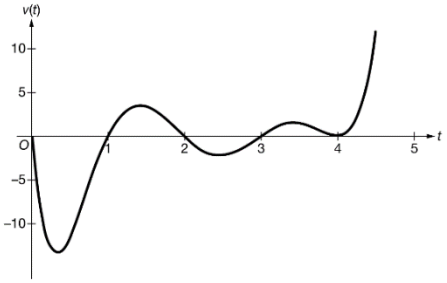
1.




The figure above shows the graph of the differentiable function $f$ for $1 \leq x \leq 11$ and the secant line through the points $(1, f(1))$ and $(11, f(11))$. For how many values of $x$ in the closed interval $[1, 11]$ does the instantaneous rate of change of $f$ at $x$ equal the average rate of change of $f$ over that interval?

A) 0 B) 2 C) 3 D) 4

2.




A particle moves along the $y$-axis. The graph of the particle's velocity $v(t)$ at time $t$ is shown above for $0 < t < 4.5$. How many times does the particle change direction over the time interval $0 < t < 4.5$?

A) 3 B) 4 C) 5 D) 8

3. A particle moves along the $x$-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 + 3t^2 - 36t + 50$. What is the total distance traveled by the particle over the time interval $0 \leq t \leq 5$?

A) 145 B) 180 C) 195 D) 233

4. A particle moves along the $y$-axis so that at time $t \geq 0$ its position is given by $y(t) = t^3 - 6t^2 + 9t$. Over the time interval $0 < t < 4$, for what values of $t$ is the speed of the particle increasing?

(A) $2 < t < 4$

(B) $3 < t < 4$ only

(C) $0 < t < 1$ and $3 < t < 4$

(D) $1 < t < 2$ and $3 < t < 4$

5. A rectangle has width $w$ inches and height $h$ inches, where the width is twice the height. Both $w$ and $h$ are functions of time $t$, measured in seconds. If $A$ represents the area of the rectangle, which of the following gives the rate of change of $A$ with respect to $t$?

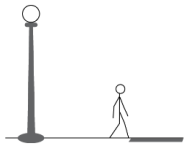
(A) $\frac{dA}{dt} = 4h \text{ in/sec}$

(B) $\frac{dA}{dt} = 3h\frac{dh}{dt} \text{ in}^2\text{/sec}$

(C) $\frac{dA}{dt} = 4h\frac{dh}{dt} \text{ in/sec}$

(D) $\frac{dA}{dt} = 4h\frac{dh}{dt} \text{ in}^2\text{/sec}$

6.



A person whose height is $M$ feet is walking away from the base of a streetlight along a straight road, as shown in the figure above. The height of the streetlight is $L$ feet. At time $t$ seconds, the person is $x$ feet from the streetlight, and the length of the person's shadow is $s$ feet. The quantities are related by the equation $\frac{1}{L}(x+s)=\frac{1}{M}s$, where $L$ and $M$ are constants. Which of the following best describes the relationship between the rate of change of $x$ with respect to time and the rate of change of $s$ with respect to time?

(A) $\frac{dx}{dt}=\frac{L}{M}s-s$

(B) $\frac{dx}{dt}=\frac{L}{M}s-\frac{ds}{dt}$

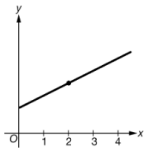
(C) $\frac{dx}{dt}=\frac{L}{M}\frac{ds}{dt}-s$

(D) $\frac{dx}{dt}=\frac{L}{M}\frac{ds}{dt}-\frac{ds}{dt}$

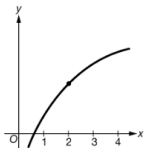
7.

The locally linear approximation of the differentiable function $f$ at $x=2$ is used to approximate the value of $f(2.3)$. The approximation at $x=2.3$ is an underestimate of the corresponding function value at $x=2.3$. Which of the following could be the graph of $f$?

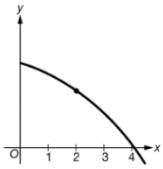
(A)



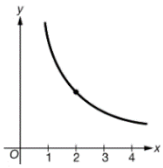

(B)




(C)




(D)




8.

Which of the following limits does not yield an indeterminate form?

(A) $\lim_{x\to 0}\frac{3x^2}{x-\sin x}$

(B) $\lim_{x\to 2}\frac{\ln\left(\frac{x}{2}\right)}{x^2-5x+6}$

(C) $\lim_{x\to \pi}\frac{x-\pi}{\cos x}$

(D) $\lim_{x\to \infty}\frac{e^{3x}}{x^{100}}$

9.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| | | | | |
| 3 | 2 | $-4$ | 1 | 2 |

Selected values of the twice-differentiable functions $f$ and $g$ and their derivatives are given in the table above. The value of $\lim_{x\to 3}\frac{x^3f(x)-54}{g(x)-1}$ is

(A) $-108$

(B) $-54$

(C) $-27$

(D) nonexistent

10. If $g(x) = \ln x$ and $f$ is a differentiable function of $x$, which of the following is equivalent to the derivative of $f(g(x))$ with respect to $x$ ?

Ⓐ $f'\left(\frac{1}{x}\right)$

Ⓑ $\frac{f'(x)}{x}$

Ⓒ $f'(\ln x)$

Ⓓ $\frac{f'(\ln x)}{x}$

11. For which of the following functions is the chain rule an appropriate method to find the derivative with respect to $x$ ?

I. $y = \sin\left(3x^2\right)$
II. $y = e^x \tan x$
III. $y = \frac{1}{8x^4 - 2x}$

Ⓐ I only

Ⓑ II only

Ⓒ III only

Ⓓ I and III only

12. Let $f$ be a differentiable function. If $h(x) = (1 + f(3x))^2$, which of the following gives a correct process for finding $h'(x)$ ?

Ⓐ $h'(x) = 2(1 + f(3x))$

Ⓑ $h'(x) = 2(1 + f(3x)) \cdot f'(3x)$

Ⓒ $h'(x) = 2(1 + f(3x)) \cdot f'(x)$

Ⓓ $h'(x) = 2(1 + f(3x)) \cdot f'(3x) \cdot 3$

13. What is the slope of the line tangent to the curve $y^3 - xy^2 + x^3 = 5$ at the point $(1, 2)$ ?

Ⓐ $\frac{1}{10}$

Ⓑ $\frac{1}{8}$

Ⓒ $\frac{5}{12}$

Ⓓ $\frac{11}{4}$

14. If $\sin(x+y) = 3x - 2y$, then $\frac{dy}{dx} =$

(A) $\frac{3-\cos(x+y)}{2}$

(B) $\frac{1-\cos(x+y)}{\cos(x+y)}$

(C) $\frac{3}{2+\cos(x+y)}$

(D) $\frac{3-\cos(x+y)}{2+\cos(x+y)}$

15. $\frac{d}{dx}\left(\sin^{-1}x\right)\Big|_{x=\frac{1}{2}} =$

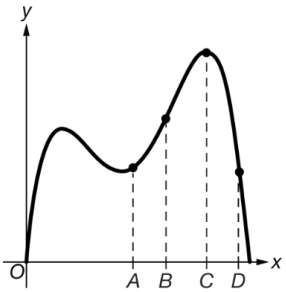
(A) $\frac{1}{1+\left(\frac{1}{2}\right)^2}$

(B) $\frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}$

(C) $\cos^{-1}\left(\frac{1}{2}\right)$

(D) $-\csc\left(\frac{1}{2}\right)\cot\left(\frac{1}{2}\right)$

16.



Graph of $f'$

The figure above shows the graph of $f'$, the derivative of the function $f$. At which of the four indicated values of $x$ is $f''(x)$ greatest?

(A) $A$

(B) $B$

(C) $C$

(D) $D$

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17. The derivative of a function $f$ is given by $f'(x) = 0.1x + e^{0.25x}$. At what value of $x$ for $x > 0$ does the line tangent to the graph of $f$ at $x$ have slope 2 ?

(A) 0.512

(B) 1.849

(C) 2.287

(D) 8.113

18. The function $f$ is given by $f(x) = 0.1x^4 - 0.5x^3 - 3.3x^2 + 7.7x - 1.99$. For how many positive values of $b$ does $\lim_{x \to b} f(x) = 2$ ?

(A) One

(B) Two

(C) Three

(D) Four

19.

$$f(x) = \begin{cases} e^{bx} & \text{for } x \le 2 \\ 1.5x + b & \text{for } x > 2 \end{cases}$$

Let $f$ be the function defined above. For what values of $b$ is $f$ continuous at $x = 2$?

(A) 0.508 only

(B) 0.647 only

(C) $-1.282$ and 0.508

(D) $-2.998$ and 0.647

20. Let $f$ be the function given by $f(x) = x + \tan\left(\frac{x}{5}\right) - 10$. The Intermediate Value Theorem applied to $f$ on the closed interval $[12, 15]$ guarantees a solution in $[12, 15]$ to which of the following equations?

(A) $f(x) = -10$

(B) $f(x) = 0$

(C) $f(x) = 4$

(D) $f(x) = 14$

21. A particle moves along a line so that at time $t$, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

(A) –5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

22. If the derivative of $f$ is given by $f'(x) = e^x - 3x^2$, at which of the following values of $x$ does $f$ have a relative maximum value?

(A) –0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

23. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) –1.58 (B) –1.63 (C) –1.67 (D) –1.89 (E) –2.33

24. The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere? (The volume $V$ of a sphere with radius $r$ is $V = \frac{4}{3}\pi r^3$.)

(A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm