AP Calculus - Chapter 2 Test 1 –Review 2

The test will be a "mini test" worth 50 points.

It will all be NO CALCULATOR.

Section 1 will be 7 multiple choice questions. Know how to estimate speed; how to set up the definition of a derivative; know how to find limits using a table or graph; know how to find limits involving sine or cosine; know your limit laws; know how to apply the squeeze theorem; know how to find finite and infinite limits in general using the steps we discussed.

Section 2 will be free response, no calc.

Know how to solve limits given a piecewise function; know how to solve limits given a graph of f and g and applying limit laws; know how to find limits given a graph and a table along with limit laws; be able to find the average rate of change given a table or function; know how to find the instantaneous rate of change and average rate of change given a table or function; know how the find the slope of a tangent given a function and a point; be able to find the normal line to a curve.

All No Calculator!!!

1. Find each limit:

$$\lim_{x\to\infty} \frac{20}{10+3e^{.04x}} = \frac{20}{10} = \boxed{2}$$

$e^{-\infty} =$ small #
0

$$\lim_{x\to0} \frac{\cos x}{\sin x - 5e^x} = \frac{1}{0-5e^0} = \boxed{-\frac{1}{5}}$$

2. $\lim_{x\to3} f(x) = 3$ and $\lim_{x\to3} \frac{g(x)}{f(x)} = 8$. What is the $\lim_{x\to3} g(x)$?

$$\frac{\lim_{x\to3} g(x)}{\lim_{x\to3} f(x)} = \frac{\lim_{x\to3} g(x)}{3} = 8$$

Thus $= \boxed{24}$

3. Suppose $f(x) \le g(x) \le h(x)$ and $\lim_{x\to-2} f(x) = \lim_{x\to-2} h(x) = -7$. Find $\lim_{x\to-2} g(x)$.

-7 -7

Thus by squeeze thm $\boxed{\lim_{x\to-2} g(x) = -7}$

4. $f(x) = \begin{cases} x+2 & for\ x>3 \\ -5x+3 & for\ x<3 \end{cases}$ Find: a) $\lim_{x\to3^-} f(x)$ b) $\lim_{x\to3^+} f(x)$ c) $\lim_{x\to3} f(x)$

$-5(3)+3$ $3+2$

$\boxed{-12}$ $= \boxed{5}$ $\boxed{\text{DNE}}$
                                         DIVERGES

5. Use the graph of f(x) and g(x) to find each:

Graph of f(x)
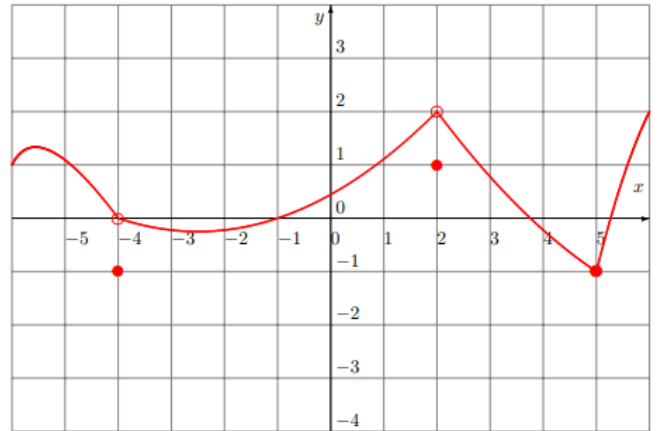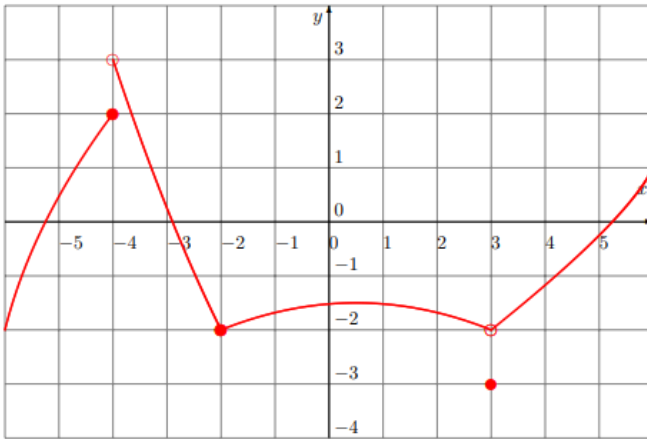
Find the following limits:

Graph of g(x)

a) $\lim_{x \to -2^-} f(x)$ b) $\lim_{x \to -2^+} f(x)$ c) $\lim_{x \to -2} f(x)$ d) $\lim_{x \to -4} f(x)$ e) $\lim_{x \to 3} f(x)$

$\boxed{-2}$ $\boxed{-2}$ $\boxed{-2}$ $\boxed{DNE}$ $\boxed{-2}$

f) f(3) g) $\lim_{x \to 2} \frac{f(x)}{g(x)} = \boxed{\frac{-1.7}{2}}$ h) $\lim_{x \to 1} \frac{f(x)+1}{g(x)} =$ j) $\lim_{x \to -4} [3f(x) + 2g(x)] = 3(2) + 2(0)$

$\boxed{-3}$

$\frac{1.5+1}{1} = \boxed{2.5}$ $= 6 + 0 = \boxed{6}$

6. Selected values of a function f(x) are shown in the table. What is the average rate of change of f over the interval [-2, 5]?

x	-4	-2	0	1	2	4	5	10
f(x)	10	5	-3	0	15	5	12	8

$AvE = \frac{f(5) - f(-2)}{5 - -2}$

$= \frac{12 - 5}{7} = \frac{7}{7} = \boxed{1}$

7. Suppose you have the following:

$$g(x) = \begin{cases} x^2 + 2x + 1 & for\ 0 \le x < 6 \\ f(x) & for\ 6 \le x \le 12 \end{cases}$$

x	6	8	10	12
f(x)	100	120	141	202

a) According to the model g, what is the average rate of change over the time interval $6 \le x \le 12$?

$\frac{f(12) - f(6)}{12 - 6} = \frac{202 - 100}{6} = \frac{102}{6} = \boxed{17}$ need table for interval

b) Use the data in the table to approximate the instantaneous rate of change at x = 11.

since we don't have a value @ f(11), need to approx by the 2 closest to it

$\frac{f(12) - f(10)}{12 - 10}$

$\frac{202 - 141}{2}$

c) Find the instantaneous rate of change at x = 2.

Need P (2, g(x)) Q (x, g(x))

P (2, 9) Q (x, x²+2x+1)

Mpq $\lim_{x \to 2} \frac{x^2 + 2x + 1 - 9}{x - 2} =$

$\frac{(x+4)(x-2)}{(x-2)} = \boxed{6}$

$= \boxed{\frac{61}{2}}$

8. Let $f(x) = 4x^2 - 6x$ and P the point $(1, -2)$.

a) Find the slope of the curve $y = f(x)$ at P.

$P(1, -2)$ $Q(x, 4x^2 - 6x)$

$m_{PQ} = \lim_{x \to 1} \dfrac{4x^2 - 6x - 2}{x - 1}$

$\lim_{x \to 1} \dfrac{4x^2 - 6x + 2}{(x-1)} = \dfrac{2(2x^2 - 3x + 1)}{(x-1)}$

$= \dfrac{2(2x-1)(x-1)}{(x-1)}$

$= \boxed{2}$

b) The equation of the tangent at P.

$\boxed{y + 2 = 2(x-1)}$

c) The equation of the normal at P.

$\hookrightarrow \perp \quad m = 2 \quad m_\perp = -\dfrac{1}{2}$

$\boxed{y + 2 = -\dfrac{1}{2}(x-1)}$